EDUDEVS

Upskilling Education

CLASS X

MATHAMATICS

RESOURCE MATERIAL

CH 1-REAL NUMBERS

LESSON PLAN

Class & Section –X
No of Periods allotted per week: 1
Duration: 40 minutes

Subject: Mathematics
MARCH & APRIL

Chapter – 1 : Real Numbers

PRE – REQUISITE KNOWLEDGE :

- Rational Numbers: Class VIII
- Number System: Class IX.

OBJECTIVES :

The students will be able to:

- describe Euclid's Division Lemma
- calculate the HCF of two positive integers using Euclid's Division Lemma/ Algorithm
- describe the Fundamental Theorem of Arithmetic.
- express a number as a product of it's prime factors to calculate the HCF and LCM of numbers.
- recall the properties of irrational numbers.
- prove that if P is prime and P divides a 2 , then P divides a, where a is a positive integer.
- prove that √2, √3, √5 are irrational numbers.

In this chapter, students will study:

- **Natural numbers:** Counting numbers are called Natural numbers. These numbers are denoted by N = {1, 2, 3, ……}
- **Whole numbers:** The collection of natural numbers along with 0 is the collection of Whole number and is denoted by W.
- **Integers:** The collection of natural numbers, their negatives along with the number zero are called Integers. This collection is denoted by Z.
- **Rational number:** The numbers, which are obtained by dividing two integers, are called Rational numbers. Division by zero is not defined. Coprime: If HCF of two numbers is 1, then the two numbers area is called relatively prime or coprime.
- **Coprime:** If HCF of two numbers is 1, then the two numbers area is called relatively prime or coprime.

BEHAVIORAL OUTCOMES :

Students will be able to:

- explain Euclid's Division Lemma and the Fundamental Theorem of Arithmetic.
- calculate the HCF of numbers using Euclid's Division Lemma and HCF & LCM of numbers using the prime factorization method.
- recall the properties of rational and irrational numbers and prove a few theorems pertaining to these numbers.

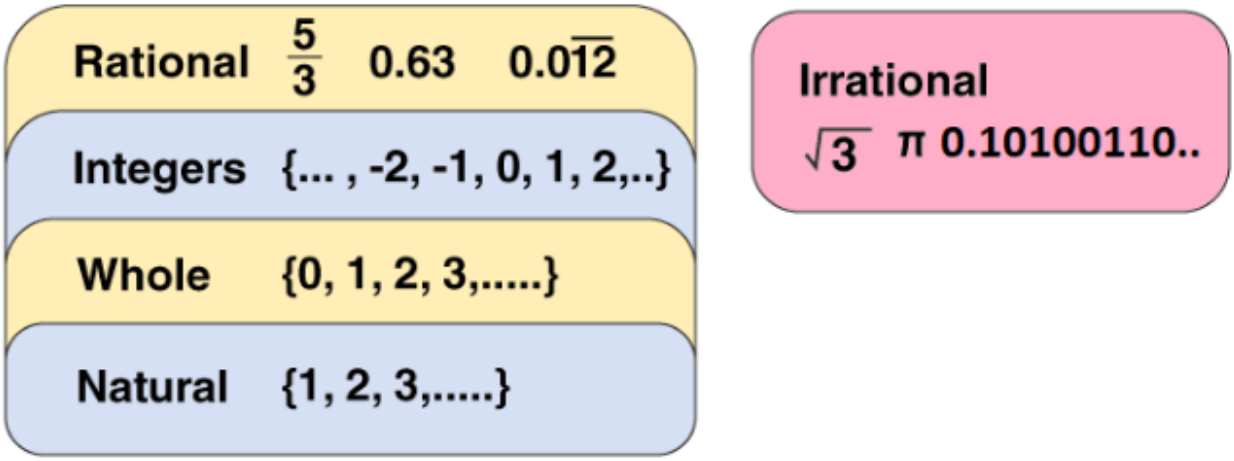
Sign of Coordinator:

Sign of the Principal:

CHAPTER SUMMARY

Real Numbers

Real numbers can be defined as the union of both rational and irrational numbers. They can be both positive or negative and are denoted by the symbol “R”. All the natural numbers, decimals and fractions come under this category.



Set of Real Numbers

The set of real numbers consists of different categories, such as natural and whole numbers, integers, rational and irrational numbers. In the table given below, all the real numbers formulas (i.e.) the representation of the classification of real numbers are defined with examples.

Category	Definition	Example
Natural Numbers	Contain all counting numbers which start from 1. N = {1, 2, 3, 4,.....}	All numbers such as 1, 2, 3, 4, 5, 6,.....
Whole Numbers	Collection of zero and natural numbers. W = {0, 1, 2, 3,.....}	All numbers including 0 such as 0, 1, 2, 3, 4, 5, 6,.....
Integers	The collective result of whole numbers and negative of all natural numbers.	Includes: -infinity (-∞),.....-4, -3, -2, -1, 0, 1, 2, 3, 4, ...+infinity (+∞)
Rational Numbers	Numbers that can be written in the form of p/q, where q≠0.	Examples of rational numbers are ½, 5/4 and 12/6 etc.
Irrational Numbers	The numbers which are not rational and cannot be written in the form of p/q.	Irrational numbers are non-terminating and non-repeating in nature like √2.

Properties of Real Numbers

The following are the four main properties of real numbers:

- Commutative property
- Associative property
- Distributive property
- Identity property

Consider “m, n and r” are three real numbers. Then the above properties can be described using m, n, and r as shown below:

Commutative Property

If m and n are the numbers, then the general form will be m + n = n + m for addition and m.n = n.m for multiplication.

- **Addition:** m + n = n + m. For example, 5 + 3 = 3 + 5, 2 + 4 = 4 + 2.
- **Multiplication:** m × n = n × m. For example, 5 × 3 = 3 × 5, 2 × 4 = 4 × 2.

Associative Property

If m, n and r are the numbers. The general form will be m + (n + r) = (m + n) + r for addition(mn) r = m (nr) for multiplication.

- Addition: The general form will be m + (n + r) = (m + n) + r. An example of additive associative property is 10 + (3 + 2) = (10 + 3) + 2.
- Multiplication: (mn) r = m (nr). An example of a multiplicative associative property is (2 × 3) 4 = 2 (3 × 4).

Distributive Property

For three numbers m, n, and r, which are real in nature, the distributive property is represented as:
m (n + r) = mn + mr and (m + n) r = mr + nr.

- Example of distributive property is: 5(2 + 3) = 5 × 2 + 5 × 3. Here, both sides will yield 25.

Identity Property

There are additive and multiplicative identities.

- **For addition:** m + 0 = m. (0 is the additive identity)
- **For multiplication:** m × 1 = 1 × m = m. (1 is the multiplicative identity)

SOLVED EXAMPLES

Example 1: Find five rational numbers between 1/2 and 3/5.

Solution:

We shall make the denominator same for both the given rational number

(1 × 5)/(2 × 5) = 5/10 and (3 × 2)/(5 × 2) = 6/10

Now, multiply both the numerator and denominator of both the rational number by 6, we have

(5 × 6)/(10 × 6) = 30/60 and (6 × 6)/(10 × 6) = 36/60

Five rational numbers between 1/2 = 30/60 and 3/5 = 36/60 are

31/60, 32/60, 33/60, 34/60, 35/60.

Example 2: Write the decimal equivalent of the following:
(i) 1/4 (ii) 5/8 (iii) 3/2

Solution:
(i) 1/4 = (1 × 25)/(4 × 25) = 25/100 = 0.25

(ii) 5/8 = (5 × 125)/(8 × 125) = 625/1000 = 0.625
(
iii) 3/2 = (3 × 5)/(2 × 5) = 15/10 = 1.5

Example 3: What should be multiplied to 1.25 to get the answer 1?

Solution:

1.25 = 125/100

Now if we multiply this by 100/125, we get

125/100 × 100/125 = 1

Euclid's Division Lemma: An Introduction

According to Euclid's Division Lemma if we have two positive integers a and b, then there exist unique integers q and r which satisfies the condition a = bq + r where 0 ≤ r < b.

The basis of the Euclidean division algorithm is Euclid's division lemma. To calculate the Highest Common Factor (HCF) of two positive integers a and b we use Euclid's division algorithm. HCF is the largest number which exactly divides two or more positive integers. That means, on dividing both the integers a and b the remainder is zero.

Euclid's Division Lemma Algorithm

Consider two numbers 78 and 980 and we need to find the HCF of these numbers. To do this, we choose the largest integer first, i.e. 980 and then according to Euclid Division Lemma, a = bq + r where 0 ≤ r < b;

980 = 78 × 12 + 44

Now, here a = 980, b = 78, q = 12 and r = 44.

Now consider the divisor 78 and the remainder 44, apply Euclid division lemma again.

78 = 44 × 1 + 34

Similarly, consider the divisor 44 and the remainder 34, apply Euclid division lemma to 44 and 34.

44 = 34 × 1 + 10

Following the same procedure again,

34 = 10 × 3 + 4

10 = 4 × 2 + 2

4 = 2 × 2 + 0

As we see that the remainder has become zero, therefore, proceeding further is not possible. Hence, the HCF is the divisor b left in the last step. We can conclude that the HCF of 980 and 78 is 2.

Let us try another example to find the HCF of two numbers 250 and 75. Here, the larger the integer is 250, therefore, by applying Euclid Division Lemma a = bq + r where 0 ≤ r < b, we have

a = 250 and b = 75

⇒ 250 = 75 × 3 + 25

By applying the Euclid's Division Algorithm to 75 and 25, we have:

75 = 25 × 3 + 0

As the remainder becomes zero, we cannot proceed further. According to the

algorithm, in this case, the divisor is 25. Hence, the HCF of 250 and 75 is 25.

Example 4: Find the HCF of 81 and 675 using the Euclidean division algorithm.

Solution: The larger integer is 675, therefore, by applying the Division Lemma a = bq + r where 0 ≤ r < b, we have

a = 675 and b = 81

⇒ 675 = 81 × 8 + 27

By applying Euclid's Division Algorithm again we have,

81 = 27 × 3 + 0

We cannot proceed further as the remainder becomes zero. According to the algorithm, in this case, the divisor is 27. Hence, the HCF of 675 and 81 is 27.

Fundamental Theorem Of Arithmetic

Fundamental Theorem of Arithmetic states that every integer greater than 1 is either a prime number or can be expressed in the form of primes. In other words, all the natural numbers can be expressed in the form of the product of its prime factors. To recall, prime factors are the numbers which are divisible by 1 and itself only. For example, the number 35 can be written in the form of its prime factors as:

35 = 7 × 5

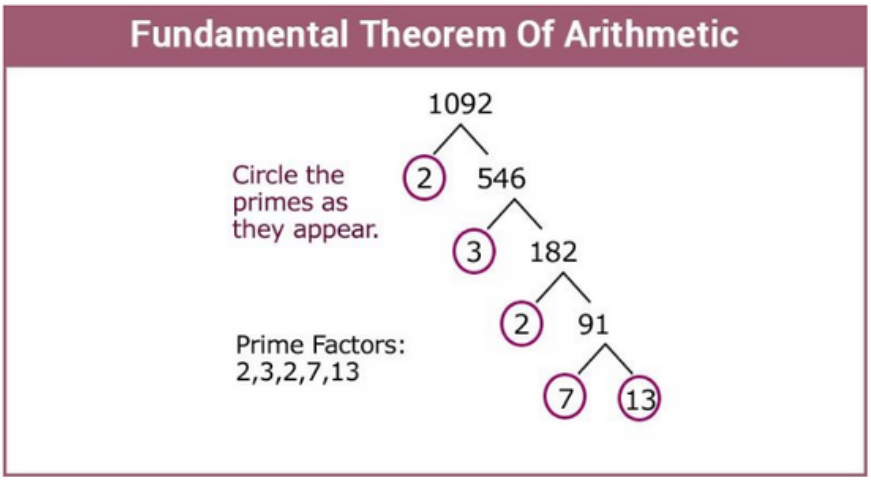
Here, 7 and 5 are the prime factors of 35

Similarly, another number 114560 can be represented as the product of its prime factors by using prime factorization method,

114560 = 27 × 5 × 179

So, we have factorized 114560 as the product of the power of its primes.

Therefore, every natural number can be expressed in the form of the product of the power of its primes. This statement is known as the F**undamental Theorem of Arithmetic,** unique factorization theorem or the unique-prime-factorization theorem.



Proof for Fundamental Theorem of Arithmetic

In Number theory, a composite number is expressed in the form of the product of primes and this factorization is unique apart from the order in which the prime factor occurs.
From this theorem we can also see that not only a composite number can be factorized as the product of their primes but also for each composite number the factorization is unique, not taking into consideration order of occurrence of the prime factors.

In simple words, there exists only a single way to represent a natural number by the product of prime factors. This fact can also be stated as:

The prime factorization of any natural number is said to be unique for except the order of their factors.

In general, a composite number “a” can be expressed as,

a = p1 p2 p3 …… pn, where p1, p2, p3 …… pn are the prime factors of a written in ascending order i.e. $p1 \leq p2 \leq p3 \ldots\ldots \leq pn$.

Writing the primes in ascending order makes the factorization unique in nature.

**Example 5: In a formula racing competition the time taken by two racing cars A and B to complete 1 round of the track is 30 minutes and 45 minutes respectively.
After how much time will the cars meet again at the starting point?**

Solution:

As the time taken by car B is more compared to that of A to complete one round therefore it can be assumed that A will reach early and both the cars will meet again when A has already reached the starting point. This time can be calculated by finding the L.C.M of the time taken by each.

$30 = 2 \times 3 \times 5$

$45 = 3 \times 3 \times 5$

The L.C.M is 90.

Thus, both cars will meet at the starting point after 90 minutes.

Example 6: An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Given,
Number of army contingent members = 616

Number of army band members = 32

If the two groups have to march in the same column, we have to find out the highest common factor between the two groups. HCF(616, 32), gives the maximum number of columns in which they can march.

By using Euclid's algorithm to find their HCF, we get,
Since, 616>32, therefore,

$616 = 32 \times 19 + 8$

Since, $8 \neq 0$, therefore, taking 32 as new divisor, we have,
$32 = 8 \times 4 + 0$

Now we have got remainder as 0, therefore, HCF (616, 32) = 8.

Hence, the maximum number of columns in which they can march is 8.

Example 7: Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solution:

Let x be any positive integer and y = 3.

By Euclid's division algorithm, then,

$x = 3q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, $x = 3q, 3q+1$ and $3q+2$

Now as per the question given, by squaring both the sides, we get,

$x2 = (3q)2 = 9q2 = 3 \times 3q2$

Let $3q2 = m$

Therefore, $x2= 3m$ ……..(1)

$x2 = (3q + 1)2 = (3q)2+12+2\times3q\times1 = 9q2 + 1 +6q = 3(3q2+2q) +1$

Substitute, $3q2+2q = m$, to get,

$x2= 3m + 1$ ……. (2)

x2= (3q + 2)2 = (3q)2+22+2×3q×2 = 9q2 + 4 + 12q = 3 (3q2 + 4q + 1)+1

Again, substitute, 3q2+4q+1 = m, to get,

x2= 3m + 1…… (3)

Hence, from equation 1, 2 and 3, we can say that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m**.**

What are Irrational Numbers?

An irrational number is a real number that cannot be expressed as a ratio of integers; for example, √2 is an irrational number. We cannot express any irrational number in the form of a ratio, such as p/q, where p and q are integers, q≠0. Again, the decimal expansion of an irrational number is neither terminating nor recurring.

The common examples of irrational numbers are pi(π=3 · 14159265…), √2, √3, √5, Euler's number (e = 2 · 718281…..), 2.010010001….,etc.

Is Pi an irrational number?

Yes, Pi (π) is an irrational number because it is neither terminating nor repeating decimal. Also, Pi is not equal to 22/7 as 22/7 is a rational number while pi is an irrational number. The value of π is 3.141592653589……..
Note- Rational numbers (Q) and Irrational numbers (P or Q') are always alternate with each other.
Therefore, 22/7 ≠ π but they are alternate or next to each other.

Sum and Product of Two Irrational Numbers

Now, let us discuss the sum and the product of irrational numbers

Product of Two Irrational Numbers

Statement: The product of two irrational numbers is sometimes rational or irrational

For example, √2 is an irrational number, but when √2 is multiplied by √2, we get the result 2, which is a rational number.

(i.e.,) √2 x √2 = 2
We know that π is also an irrational number, but if π is multiplied by π, the result is π2, which is also an irrational number.

(i.e..) π x π = π2
It should be noted that while multiplying the two irrational numbers, it may result in an irrational number or a rational number.

Sum of Two Irrational Numbers

Statement: The sum of two irrational numbers may be rational or irrational.
Like the product of two irrational numbers, the sum of two irrational numbers will also result in a rational or irrational number.

For example, if we add two irrational numbers, say $3\sqrt{2}+ 4\sqrt{3}$, a sum is an irrational number.
But, let us consider another example, $(3+4\sqrt{2}) + (-4\sqrt{2})$, the sum is 3, which is a rational number.
So, we should be very careful while adding and multiplying two irrational numbers, because it might result in an irrational number or a rational number.

Irrational Number Theorem and Proof

The following theorem is used to prove the above statement

Theorem: Given p is a prime number and a^2 is divisible by p, (where a is any positive integer), then it can be concluded that p also divides a.

Proof: Using the Fundamental Theorem of Arithmetic, the positive integer can be expressed in the form of the product of its primes as:

$a = p_1 \times p_2 \times p_3 \ldots\ldots \times p_n$ …..(1)

Where, $p_1, p_2, p_3, \ldots\ldots, p_n$ represent all the prime factors of a.

Squaring both the sides of equation (1),

$a^2 = (p_1 \times p_2 \times p_3 \ldots\ldots \times p_n) (p_1 \times p_2 \times p_3 \ldots\ldots \times p_n)$

$\Rightarrow a^2 = (p_1)^2 \times (p_2)^2 \times (p_3)^2 \ldots\ldots \times (p_n)^2$

According to the Fundamental Theorem of Arithmetic, the prime factorization of a natural number is unique, except for the order of its factors.

The only prime factors of a^2 are $p_1, p_2, p_3 \ldots\ldots, p_n$. If p is a prime number and a factor of a^2, then p is one of $p_1, p_2, p_3 \ldots\ldots, p_n$. So, p will also be a factor of a.

Hence, if a^2 is divisible by p, then p also divides a.

Now, using this theorem, we can prove that $\sqrt{2}$ is irrational.

How to Find an Irrational Number?

Let us find the irrational numbers between 2 and 3.
We know the square root of 4 is 2; √4 =2

and the square root of 9 is 3; √9 = 3

Therefore, the number of irrational numbers between 2 and 3 are √5, √6, √7, and √8, as these are not perfect squares and cannot be simplified further. Similarly, you can also find irrational numbers, between any other two perfect square numbers.

Another case:

Let us assume a case of √2. Now, how can we find if √2 is an irrational number? Suppose √2 is a rational number. Then, by the definition of rational numbers, it can be written that,
√ 2 =p/q ……(1)

Where p and q are co-prime integers and q ≠ 0 (Co-prime numbers are those numbers whose common factor is 1).

Squaring both sides of equation (1), we have

2 = p2/q2

⇒ p2 = 2 q 2 ……. (2)

From the theorem stated above, if 2 is a prime factor of p2, then 2 is also a prime factor of p.

So, p = 2 × c, where c is an integer.

Substituting this value of p in equation (3), we have

(2c)2 = 2 q 2

⇒ q2 = 2c 2

This implies that 2 is a prime factor of q2 also. Again from the theorem, it can be said that 2 is also a prime factor of q.

According to the initial assumption, p and q are co-primes but the result obtained above contradicts this assumption as p and q have 2 as a common prime factor other than 1. This contradiction arose due to the incorrect assumption that √2 is rational.

So, root 2 is irrational.

Similarly, we can justify the statement discussed in the beginning that if p is a prime number, then √ p is an irrational number. Similarly, it can be proved that for any prime number p,√ p is irrational.

Example 8: Which of the following are Rational Numbers or Irrational Numbers?
2, -.45678…, 6.5, √ 3, √ 2

Solution:
Rational Numbers – 2, 6.5 as these have terminating decimals.
Irrational Numbers – -.45678…, √ 3, √ 2 as these have a non-terminating non-repeating decimal expansion.

Example 9: Check if the below numbers are rational or irrational.
2, 5/11, -5.12, 0.31

Solution: Since the decimal expansion of a rational number either terminates or repeats
So, 2, 5/11, -5.12, 0.31 are all rational numbers.

Example 10: Express each number as a product of its prime factors:
(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429

Solution:

(i) 140
By taking the LCM of 140, we will get the product of its prime factor.
Therefore, 140 = 2 × 2 × 5 × 7 × 1 = 22×5×7

(ii) 156
By Taking the LCM of 156, we will get the product of its prime factor.
Hence, 156 = 2 × 2 × 13 × 3 × 1 = 22× 13 × 3

(iii) 3825
By taking the LCM of 3825, we will get the product of its prime factor.
Hence, 3825 = 3 × 3 × 5 × 5 × 17 × 1 = 32×52×17

(iv) 5005
By Taking the LCM of 5005, we will get the product of its prime factor.
Hence, 5005 = 5 × 7 × 11 × 13 × 1 = 5 × 7 × 11 × 13

(v) 7429
By taking the LCM of 7429, we will get the product of its prime factor.
Hence, 7429 = 17 × 19 × 23 × 1 = 17 × 19 × 23

Example 11. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Solution: By the definition of composite number, we know, if a number is composite, then it means it has factors other than 1 and itself. Therefore, for the given expression;

7 × 11 × 13 + 13

Taking 13 as common factor, we get,

=13(7×11×1+1) = 13(77+1) = 13×78 = 13×3×2×13

Hence, 7 × 11 × 13 + 13 is a composite number.

Now let's take the other number,
7 × 6 × 5 × 4 × 3 × 2 × 1 + 5
Taking 5 as a common factor, we get,
=5(7×6×4×3×2×1+1) = 5(1008+1) = 5×1009
Hence, 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is a composite number.

ASSIGNMENT FOR CLASS WORK

Q1. Which is the smallest composite number?

Q2. Prove that any positive odd integer is of the form 6x + 1, 6x + 3, or 6x + 5.

Q.3: Evaluate 2 + 3 × 6 – 5.

Q.4: What is the product of a non-zero rational number and an irrational number?

Q.5:Can every positive integer be represented as 4x + 2 (where x is an integer)?

Q6. Find the HCF of 52 and 117 and express it in form 52 x + 117 y.

Q.7:Prove that x2 – x is divisible by 2 for all positive integer x.

Q.8 Find the smallest number which when increased by 20 is exactly divisible by 90 and 144.

Q.9 Find the smallest number which leaves remainder 8 and 12 when divided by 28 and 32 respectively

Q.10 Find the smallest number which leaves remainder 8 and 12 when divided by 28 and 32 respectively

Q.11 Floor of a room is to be fitted with square marble tiles of the largest possible size. The size of the room is 10 m × 7 m. What should be the size of tiles required that has to be cut and how many such tiles are required?

Q.12. Use Euclid's division algorithm to find the HCF of:
i. 135 and 225
ii. 196 and 38220
iii. 867 and 255

Q.13. Use Euclid's division algorithm to find the HCF of:
i. 135 and 225
ii. 196 and 38220
iii. 867 and 255

Q.14. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
(i) 26 and 91
(ii) 510 and 92
(iii) 336 and 54

Q.15 Prove that 3 + 2√5 + is irrational.

WORKSHEET FOR ASSESSMENT

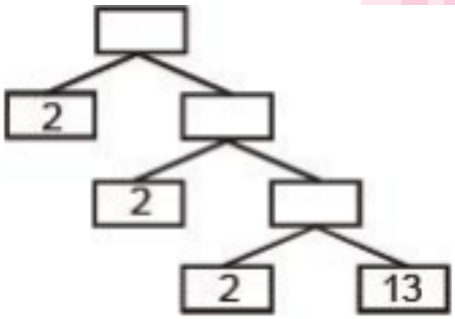
Q1. If m and n are odd positive integers, then m2 + n2 is even, but not divisible by 4. Justify.

Q2. If HCF (6, a) = 2 and LCM (6, a) = 60, then find a.

Q3. If HCF (6, a) = 2 and LCM (6, a) = 60, then find a.

Q4. If n is any prime number and a2 is divisible by n, then n will also divide a. Justify.

Q5. Find the missing numbers in prime factors tree.



Q6. Find the greatest number of 5 digits exactly divisible by 12, 15 and 36..

Q7. If the HCF of 408 and 1032 is expressible in the form 1032 p – 408 × 5 find p.

Q8. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Q9. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Q10. Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21
(ii) 17, 23 and 29
(iii) 8, 9 and 25

Q11. Given that HCF (306, 657) = 9, find LCM (306, 657).

Q12. Check whether 6n can end with the digit 0 for any natural number n..

Q13. Prove that the following are irrationals:
(i) 1/√2
(ii) 7√5
(iii) 6 + √2

CH 2 -POLYNOMIALS

LESSON PLAN

Class & Section –IX **Subject: Mathematics**

No of Periods allotted per week: 1 **MARCH & APRIL**
Duration: 40 minutes

Chapter – 2: Polynomials

PRE – REQUISITE KNOWLEDGE :

This lesson requires basic knowledge of
1. Basic knowledge of the word Geometry
2. Knowledge of basic term used in geometry e.g. point, line, straight line, surface, plane surface

OBJECTIVES :

1. Basic concepts and definitions related to the topic polynomials.
2. Explanation of remainder and factor theorem.
3. Relationships between zeroes and coefficients of quadratic polynomials and cubic polynomials.
4. Explanation of division algorithm.
5. Method of finding all zeroes of cubic and bi-quadratic polynomials.

Learning OUTCOMES :

1. After studying this lesson students will be able to explain the relationship between zeroes and coefficients.
2. Students should know the method of division of one polynomial with another and the division algorithm.
3. Students should be able to factorize the quadratic, cubic and bi-quadratic polynomials and become able to find their zeroes.

CHAPTER SUMMARY

What is a Polynomial?

Polynomial is made up of two terms, namely Poly (meaning “many”) and Nominal (meaning “terms.”). A polynomial is defined as an expression which is composed of variables, constants and exponents, that are combined using mathematical operations such as addition, subtraction, multiplication and division (No division operation by a variable). Based on the number of terms present in the expression, it is classified as monomial, binomial, and trinomial. Examples of constants, variables and exponents are as follows:

- Constants. Example: 1, 2, 3, etc.
- Variables. Example: g, h, x, y, etc.
- Exponents: Example: 5 in x5 etc.

Standard Form of a Polynomial

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots + a_1x + a_0$$

Where a_n, a_{n-1}, a_{n-2}, ……, a_1, a_0 are called coefficients of x^n, x^{n-1}, x^{n-2}, ……, x and constant term respectively and it should belong to real number ($\in R$).

Degree of a Polynomial

The degree of a polynomial is defined as the highest exponent of a monomial within a polynomial. Thus, a polynomial equation having one variable which has the largest exponent is called a degree of the polynomial.

Polynomial	Degree	Example
Zero Polynomial	Not Defined	6
Constant	0	$P(x) = 6$
Linear Polynomial	1	$P(x) = 3x+1$
Quadratic Polynomial	2	$P(x) = 4x^2+1x+1$
Cubic Polynomial	3	$P(x) = 6x^3+4x^2+3x+1$
Quartic Polynomial	4	$P(x) = 6x^4+3x^3+3x^2+2x+1$

Types of Polynomials

Depending upon the number of terms, polynomials are divided into the following categories:

CHAPTER SUMMARY

- **Monomial**
- **Binomial**
- **Trinomial**
- **Polynomial containing 4 terms (Quadronomial)**
- **Polynomial containing 5 terms (pentanomial)**
  and so on …

Monomial

A monomial is an expression which contains only one term. For an expression to be a monomial, the single term should be a non-zero term. A few examples of monomials are:

- 5x
- 3
- 6a4
- -3xy

Binomial

- A binomial is a polynomial expression which contains exactly two terms. A binomial can be considered as a sum or difference between two or more monomials. A few examples of binomials are:
- – 5x+3,
- 6a4 + 17x
- xy2+xy

Trinomial

A trinomial is an expression which is composed of exactly three terms. A few examples of trinomial expressions are:

- – 8a4+2x+7
- 4x2 + 9x + 7

Properties

Some of the important properties of polynomials along with some important polynomial theorems are as follows:

Property 1: Division Algorithm

If a polynomial P(x) is divided by a polynomial G(x) results in quotient Q(x) with remainder R(x), then,
$P(x) = G(x) \cdot Q(x) + R(x)$
Where R(x)=0 or the degree of R(x) < the degree of G(x)

Property 2: Bezout's Theorem

Polynomial P(x) is divisible by binomial (x – a) if and only if P(a) = 0.

Property 3: Remainder Theorem

If P(x) is divided by (x – a) with remainder r, then P(a) = r.

Property 4: Factor Theorem

A polynomial P(x) divided by Q(x) results in R(x) with zero remainders if and only if Q(x) is a factor of P(x).

Property 5: Intermediate Value Theorem

If P(x) is a polynomial, and $P(x) \neq P(y)$ for $(x < y)$, then P(x) takes every value from P(x) to P(y) in the closed interval [x, y].

Property 6

The addition, subtraction and multiplication of polynomials P and Q result in a polynomial where,
$\text{Degree}(P \pm Q) \leq \text{Degree}(P \text{ or } Q)$
$\text{Degree}(P \times Q) = \text{Degree}(P) + \text{Degree}(Q)$
Property 7

Property 7

If a polynomial P is divisible by a polynomial Q, then every zero of Q is also a zero of P.

Property 8

If a polynomial P is divisible by two co-prime polynomials Q and R, then it is divisible by $(Q \cdot R)$.

Property 9

If $P(x) = a0 + a1x + a2x2 + \ldots\ldots + anxn$ is a polynomial such that $\deg(P) = n \geq 0$ then, P has at most "n" distinct roots.

Property 10: Descartes' Rule of Sign

The number of positive real zeroes in a polynomial function P(x) is the same or less than by an even number as the number of changes in the sign of the coefficients. So, if there are "K" sign changes, the number of roots will be "k" or "(k − a)", where "a" is some even number.

Property 11: Fundamental Theorem of Algebra

Every non-constant single-variable polynomial with complex coefficients has at least one complex zero.

Property 12

If P(x) is a polynomial with real coefficients and has one complex zero (x = a − bi), then x = a + bi will also be a zero of P(x). Also, x2 − 2ax + a2 + b2 will be a factor of P(x).

Polynomial Functions

A polynomial function is an expression constructed with one or more terms of variables with constant exponents. If there are real numbers denoted by a, then function with one variable and of degree n can be written as:

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-2}x^2 + a_{n-1}x + a_n$$

Solving Polynomials

Any polynomial can be easily solved using basic algebra and factorization concepts. While solving the polynomial equation, the first step is to set the right-hand side as 0. The explanation of a polynomial solution is explained in two different ways:

- Solving Linear Polynomials
- Solving Quadratic Polynomials

Example: Solve 3x – 9

Solution:

First, make the equation as 0. So,
3x – 9 = 0
⇒ 3x = 9
⇒ x = 9/3
Or, x = 3.
Thus, the solution of 3x-9 is x = 3.

Example: Solve 3x2 – 6x + x3 – 18

Solution:

First, arrange the polynomial in the descending order of degree and equate to zero.

⇒ x3 + 3x2 -6x – 18 = 0

Now, take the common terms.

x2(x+3) – 6(x+3) =0

⇒ (x2-6)(x+3)=0

So, the solutions will be x =-3 or

x2 = 6

Or, x = ±√6

Example: Find the sum of two polynomials: $5x^3+3x^2y+4xy-6y^2$, $3x^2+7x^2y-2xy+4xy^2-5$

Solution:

First, combine the like terms while leaving the unlike terms as they are. Hence,

$(5x^3+3x^2y+4xy-6y^2)+(3x^2+7x^2y-2xy+4xy^2-5)$

$= 5x^3+3x^2+(3+7)x^2y+(4-2)xy+4xy^2-6y^2-5$

$= 5x^3+3x^2+10x^2y+2xy+4xy^2-6y^2-5$

Example: Solve $(6x-3y)\times(2x+5y)$

Solution:

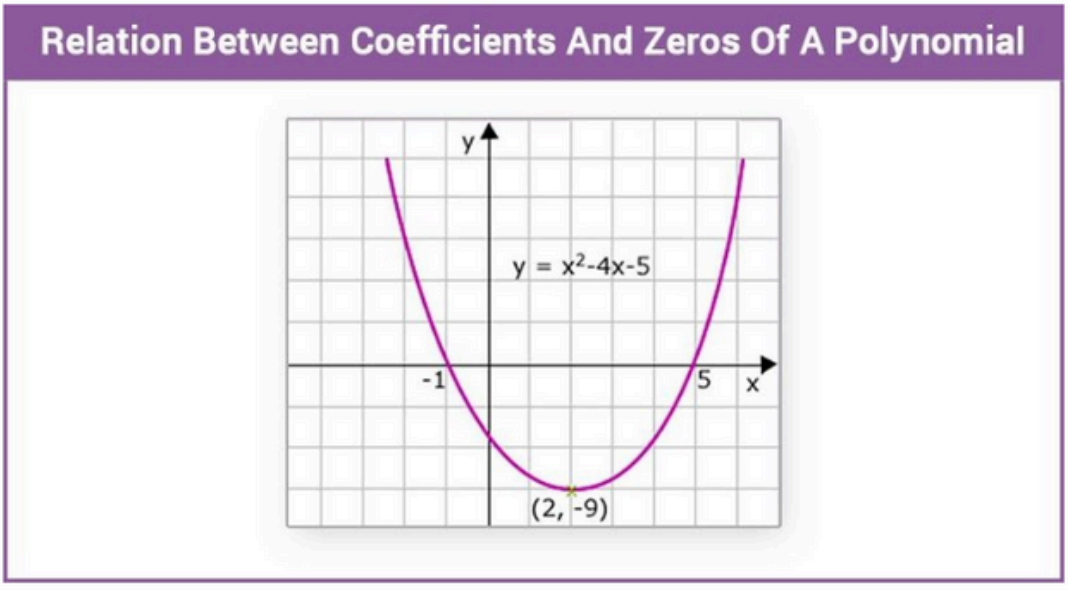
$\Rightarrow 6x \times(2x+5y)-3y \times (2x+5y)$ ———- Using distributive law of multiplication

$\Rightarrow (12x^2+30xy) - (6yx+15y^2)$ ———- Using distributive law of multiplication

$\Rightarrow 12x^2+30xy-6xy-15y^2$ ——————— as $xy = yx$

Thus, $(6x-3y)\times(2x+5y)=12x^2+24xy-15y^2$

Relation Between Zeros and Coefficient of a Polynomial



A real number say “a” is a zero of a polynomial P(x) if P(a) = 0. The zero of a polynomial is clearly explained using the Factor theorem. If “k” is a zero of a polynomial P(x), then (x-k) is a factor of a given polynomial. The relation between the zeros and the coefficients of a polynomial is given below:

Linear Polynomial

The linear polynomial is an expression, in which the degree of the polynomial is 1. The linear polynomial should be in the form of ax+b. Here, “x” is a variable, “a” and “b” are constant.
The polynomial P(x) is ax+b, then the zero of a polynomial is -b/a = – constant term/coefficient of x)

Quadratic Polynomial

The Quadratic polynomial is defined as a polynomial with the highest degree of 2. The quadratic polynomial should be in the form of ax2 + bx + c. In this case, a ≠ 0. Let say α and β are the two zeros of a polynomial, then
The sum of zeros, α + β is -b/a = – Coefficient of x/ Coefficient of x2
The product of zeros, αβ is c/a = Constant term / Coefficient of x2

Cubic Polynomial

The cubic polynomial is a polynomial with the highest degree of 3. The cubic polynomial should be in the form of ax3 + bx2 + cx + d, where a ≠ 0. Let say α, β, and γ are the three zeros of a polynomial, then
The sum of zeros, α + β + γ is -b/a = – Coefficient of x2/ coefficient of x3
The sum of the product of zeros, αβ+ βγ + αγ is c/a = Coefficient of x/Coefficient of x3
The product of zeros, αβγ is -d/a = – Constant term/Coefficient of x3

Example: Evaluate the sum and product of zeros of the quadratic polynomial 4x2 – 9.

Solution:
Given quadratic polynomial is 4x2 – 9.
4x2 – 9 can be written as 2x2 – 33, which is equal to (2x+3)(2x-3).
To find the zeros of a polynomial, equate the above expression to 0
(2x+3)(2x-3) = 0
2x+3 = 0
2x = -3
X = -3/2
Similarly, 2x-3 = 0,
2x = 3
x = 3/2
Therefore, the zeros of a given quadratic polynomial is 3/2 and -3/2.
Finding the sum and product of a polynomial:
The sum of the zeros = (3/2)+ (-3/2) = (3/2)-(3/2) = 0
The product of zeros = (3/2).(-3/2) = -9/4.

Division of a monomial by another monomial

Consider the algebraic expression 40x2 is to be divided by 10x then 40x2/10x = (2×2×5×2×x×x)/(2×5×x)

Here, 2, 5 and x are common in both the numerator and the denominator. Hence, 40x2/10x = 4x

SOLVED EXAMPLES

Q1. Find the degree of the polynomial P(x) = 6s4+ 3x2+ 5x +19

Q2. What is the value of ‘a’ if degree of polynomial, x3 + xa-4 + x2 + 1, is 4?

ASSIGNMENT FOR CLASS WORK

Q.1: How many dimensions do solids, points and surfaces have?

Q.2: What is the shape of a pyramid's base?

Q.3: If a + b =10 and a = c, then prove that c + b =10.

Q.4:Can two distinct intersecting lines be parallel to each other at the same time? Justify.

Q.5:Read the following sentence and mention which of Euclid's axiom is followed: "X's salary is equal to Y's salary. Due to the recession, the salaries of X and y are reduced to half. Now the final salary of X will still be equal to Y.

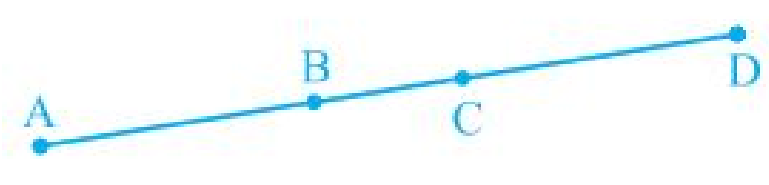
Q6.Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) parallel lines
(ii) perpendicular lines
(iii) line segment
(iv) radius of a circle
(v) square

Q7.Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) parallel lines
(ii) perpendicular lines
(iii) line segment
(iv) radius of a circle
(v) square

Q8.Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

Q9. If a point C lies between two points A and B such that AC = BC, then prove tha AC = ½ AB. Explain by drawing the figure.

Q10.if AC = BD, then prove that AB = CD.



WORKSHEET FOR ASSESSMENT

Q1 Through two points
(i) A unique line can be drawn
(ii) No line can be drawn
(iii) More than one line can be drawn

Q2.A mathematical statement whose truth has been logically established is called
(i) an axiom
(ii) a postulate
(iii) a theorem

Q3.Euclid's second axiom is
(i) 'if equals be subtracted from equals the remainders are equals'
(ii) 'the things which are equal to the same thing are equal to one another'
(iii) 'if equals be added to equals their wholes are equals'

Q4. Euclid's fifth postulate is
(i) 'the whole is greater than the part'
(ii) 'if a straight line falling on two straight lines make the interior angles on the same side of it taken together less than the right angles then the two straight lines if produced indefinitely meet on that side on which sum of angles is less than two right angles'
(iii) 'all right angles are equal to one another'?

Q5. Through a fixed point
(i) a unique line can be drawn
(ii) No line can be drawn
(iii) More than one line can be drawn

Q6.Number of line segments required to form a closed figure
(i) 2
(ii) 3
(iii)4

Q7Two lines having a common point is called
(i) parallel lines
(ii) intersecting lines
(iii) coincident lines

Q8. Euclid arranged all known work in the field of mathematics in his treatise called
(i) Elements
(ii) Axioms
(iii) Postulates

Q9. The things which are double the same thing are
(i) halves of the same thing
(ii) double of the same thing
(iii) equals

CH 6-LINES AND ANGLES

LESSON PLAN

Class & Section –IX

Subject: Mathematics
MARCH & APRIL

Chapter – 6 : Lines and Angles

PRE – REQUISITE KNOWLEDGE :

This lesson requires
1. Basic knowledge of angles and types of angles.
2. Knowledge of linear pair of angles and parallel lines.
3. Knowledge of angle sum property in a triangle.

OBJECTIVES :

Students would be able to identify the properties of the angles formed when 2 lines intersect each other and when a line intersects 2 or more parallel lines at distinct points.Students will also be able to use these properties to prove some statements, thereby inculcating competencies like collaboration, critical thinking and creativity

Learning OUTCOMES :

KNOWLEDGE- Students will develop the ability tounderstand

1. Intersecting lines and non-intersecting lines.
2. Pairs of angles.
3. Parallel lines and a transversal.
4. Lines parallel to the same line.
5. Angle sum property of a triangle.

SKILLS and COMPETENCIES-Students would be able to

1. Solve and analyze geometrical problems
2. Solve problems related to adjacent angles and linear pair.
3. Critically think anduse the concept of various angles formed when a transversal intersects 2 parallel lines and their properties.
4. Apply angle sum property of a triangle to solve various problems.
5. Solve complex questions based on the topics
6. Use analytical skills to visualize the given scenario and use the concepts learnt in everyday problems.

Sign of Coordinator:

Sign of the Principal:

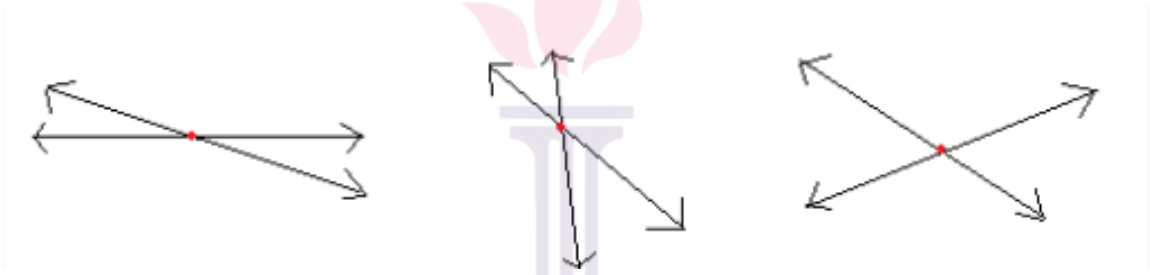
CHAPTER SUMMARY

Intersecting Lines

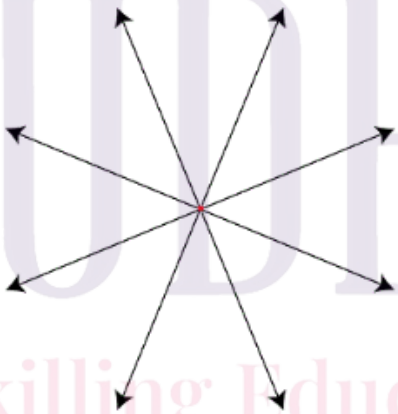
Two or more lines which share exactly one common point are called intersecting lines. This common point exists on all these lines and is called the point of intersection. It is to be noted that:

- The intersecting lines meet at one, and only one point, no matter at what angle they meet.
- No two straight lines can meet at more than one point.
- The lines that meet at more than one point are not straight lines. At least one of them is a curve.

Consider these lines. They are two intersecting lines. The point of intersection is P.



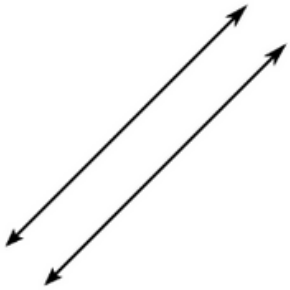
The figure above also shows intersecting lines at different angles.



Non-intersecting Lines

Two or more lines that do not intersect each other are called non-intersecting lines. It is to be noted that:

- Non-intersecting lines can never meet.
- They are also known as the parallel lines.
- They are always at the same distance from one another. This is called the distance between two parallel lines.



The figures above show non-intersecting or parallel lines.

Linear pair of angles

Linear pair of angles are formed when two lines intersect each other at a single point. The angles are said to be linear if they are adjacent to each other after the intersection of the two lines. The sum of angles of a linear pair is always equal to 180°. Such angles are also known as supplementary angles. The adjacent angles are the angles which have a common vertex. Hence, here as well the linear angles have a common vertex. Also, there will be a common arm which represents both the angles. A real-life example of a linear pair is a ladder which is placed against a wall, forms linear angles at the ground.

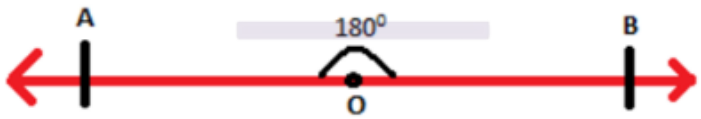
Explanation for Linear Pair of Angles

When the angle between the two lines is 180°, they form a straight angle. A straight angle is just another way to represent a straight line. A straight line can be visualized as a circle with an infinite radius. A line segment is any portion of a line which has two endpoints. Also, a portion of any line with only one endpoint is called a ray.

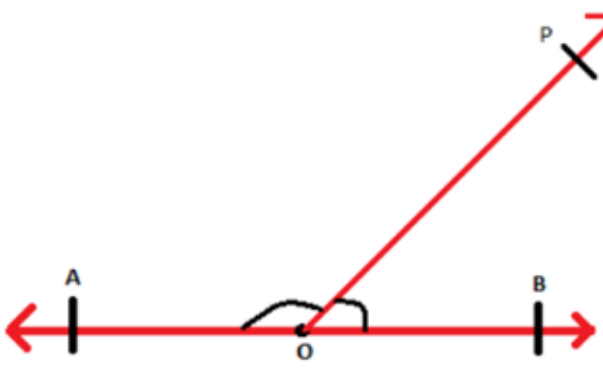
A line segment with A and B as two endpoints is represented as $\overline{AB}$.

The figure shown below represents a line segment AB and the two arrows at the end indicate a line.

If a point O is taken anywhere on the line segment AB as shown, then the angle between the two line segments AO and OB is a straight angle i.e. 180°.



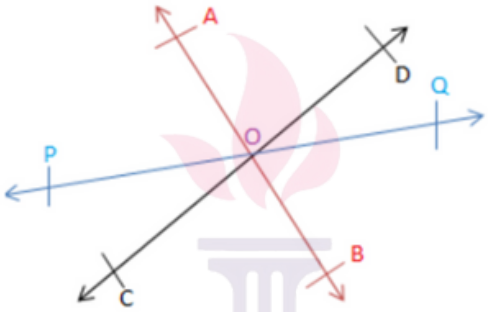
Consider a ray $\overrightarrow{OP}$ stand on the line segment $\overline{AB}$ as shown:



The angles which are formed at O are ∠POB and ∠POA. It is known that the angle between the two line segments AO and OB is 180°. therefore, the angles ∠POB and ∠POA add up to 180°.
Thus, ∠POB + ∠POA = ∠AOB = 180°
∠POB and ∠POA are adjacent to each other and when the sum of adjacent angles is 180° then such angles form linear pair of angles.
The above discussion can be stated as an axiom.

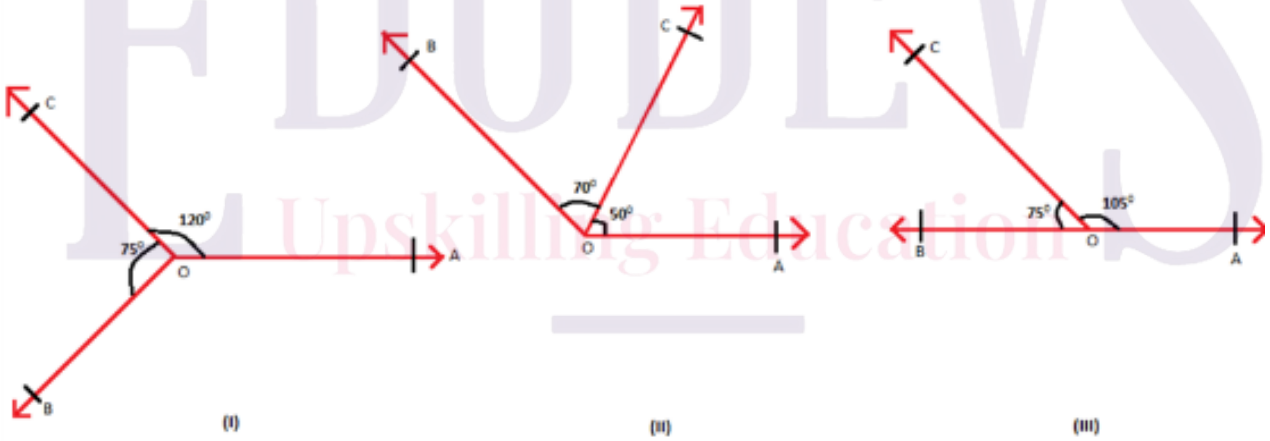
Axioms

Axiom 1: If a ray stands on a line then the adjacent angles form a linear pair of angles.



n the figure above, all the line segments pass through the point O as shown. As the ray OA lies on the line segment CD, angles ∠AOD and ∠AOC form a linear pair. Similarly, ∠QOD and ∠POD form a linear pair and so on.
The converse of the stated axiom is also true, which can also be stated as the following axiom.

Axiom 2: If two angles form a linear pair, then uncommon arms of both the angles form a straight line.



In the figure shown above, only the last one represents a linear pair, as the sum of the adjacent angles is 180°. Therefore, AB represents a line. The other two pairs of angles are adjacent but they do not form a linear pair. They do not form a straight line.
The two axioms mentioned above form the Linear Pair Axioms and are very helpful in solving various mathematical problems.

Example

Suppose two angles ∠AOC and ∠ BOC form a linear pair at point O in a line segment AB. If the difference between the two angles is 60°. Then find both the angles.

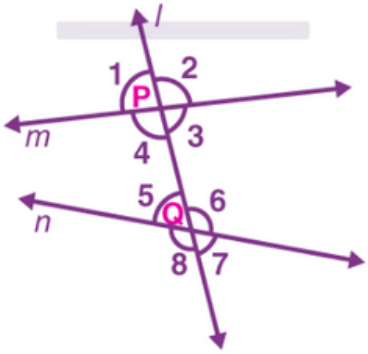
Solution: Given, ∠AOC and ∠ BOC form a linear pair
So, ∠AOC + ∠ BOC =180° .....(1)
Also given,
∠AOC − ∠ BOC = 60° .....(2)
Adding eq. 1 and 2, we get;
2∠AOC = 180° + 60° = 240°
∠AOC = 240°/2 = 120°
Now putting the value of ∠AOC in equation 1, we get;
∠BOC = 180° − ∠AOC = 180° − 120°
∠BOC = 60°

Parallel lines

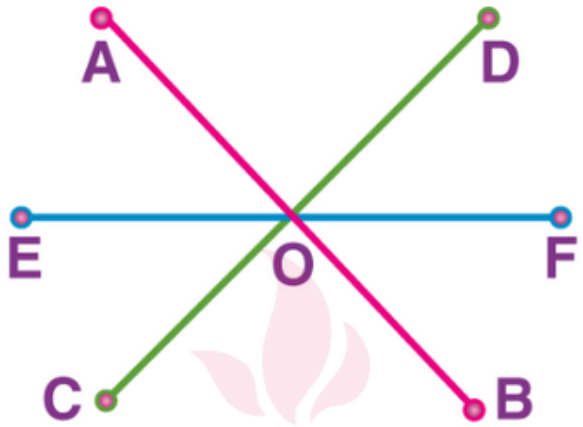
Two lines are said to be parallel when they do not intersect each other. We can also say that two lines that run along and meet at infinity are called parallel lines

Transversals

When a line intersects two lines at distinct points, it is called a transversal. In the below figure, line l intersects a and b at two distinct points, P and Q. Therefore, line l is the transversal line.

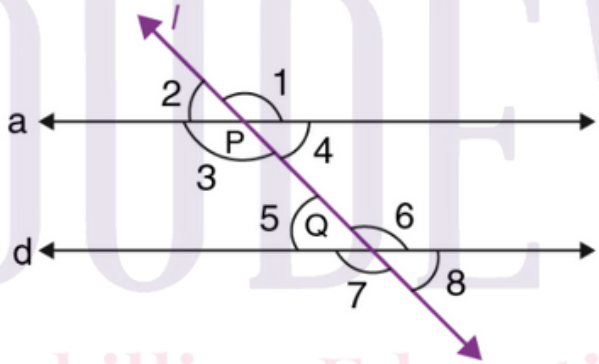


In the figure below, the line EF is not a transversal even if it intersects two lines AB and CD at O because it does not cut both lines at two distinct points. For a line to be transversal, it must cross two or more lines at separate points.



Angle Relationship Between the Parallel Lines and Transversal

Various angle pairs are formed when a transversal intersects two or more parallel lines. Let us quickly recapitulate the angle relationships for the parallel lines cut by a transversal. Let a and d be two parallel lines intersected by the transversal l at the points P and Q, as shown in the figure below.

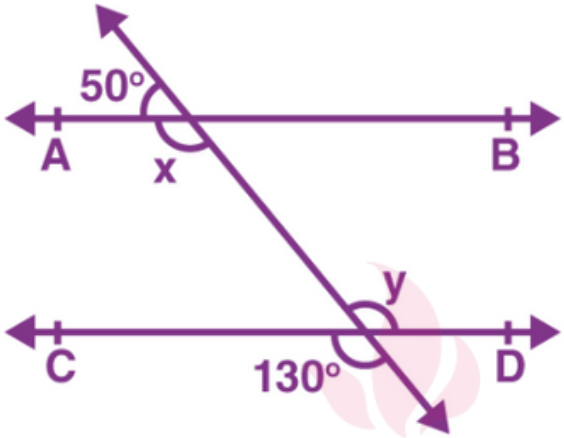


Now, we can write different pairs of angles and their corresponding relationships.

Pair of angles formed	Angle relationships
Pairs of Corresponding angles	$\angle 1 = \angle 6, \angle 4 = \angle 8, \angle 2 = \angle 5, \angle 3 = \angle 7$
Pairs of Alternate interior angles	$\angle 4 = \angle 5, \angle 3 = \angle 6$
Pairs of Alternate exterior angles	$\angle 1 = \angle 7, \angle 2 = \angle 8$
Pairs of Interior angles on the same side of the transversal	$\angle 3 + \angle 5 = 180°, \angle 4 + \angle 6 = 180°$
Pairs of Vertically opposite angles	$\angle 1 = \angle 3, \angle 2 = \angle 4, \angle 7 = \angle 6, \angle 8 = \angle 5$

Solved Example

Question: Observe the figure given below:



Find the missing angles.

Solution:
In the given figure, AB and CD are parallel lines and these are cut by a transversal line at two different points.
Here, x and 130° are the pair of corresponding angles.
So, x = 130°
Also, y and 130° are vertically opposite angles.
Thus, y = 130°

Parallel lines

Two lines are said to be parallel when they do not meet at any point in a plane. Lines which do not have a common intersection point and never cross path with each other are parallel to each other. The symbol for showing parallel lines is '||'.
Two lines which are parallel are represented as:

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

This means that line AB is parallel to CD.
The perpendicular distance between the two parallel lines is always constant.

Parallel Lines

In the figure shown above, the line segments PQ and RS represent two parallel lines as they have no common intersection point in the given plane. Infinite parallel lines can be drawn parallel to lines PQ and RS in the given plane.

Parallel Lines Axioms and Theorems

Go through the following axioms and theorems for the parallel lines.

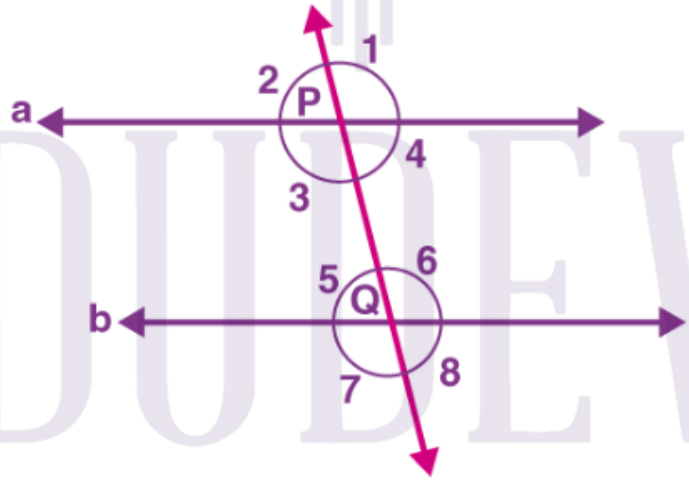
Corresponding Angle Axiom

Go through the following axioms and theorems for the parallel lines.

If two lines which are parallel are intersected by a transversal then the pair of corresponding angles are equal.

$\angle 1=\angle 6$, $\angle 4=\angle 8$, $\angle 2=\angle 5$ and $\angle 3=\angle 7$

The converse of this axiom is also true according to which if a pair of corresponding angles are equal then the given lines are parallel to each other.



Parallel Lines

Theorem 1

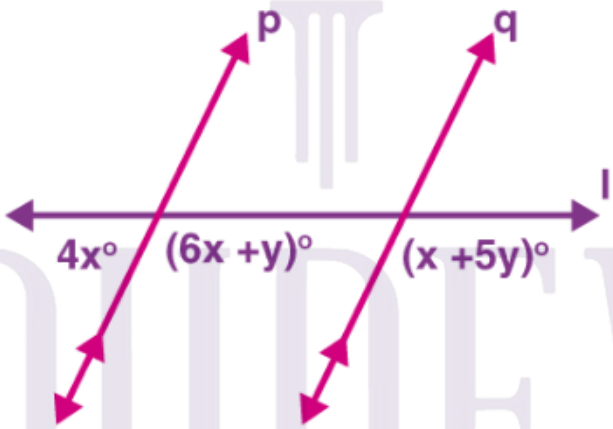
If two lines which are parallel are intersected by a transversal then the pair of alternate interior angles are equal.
From Fig. 3: $\angle 4=\angle 5$ and $\angle 3=\angle 6$
Proof: As, $\angle 4=\angle 2$ and $\angle 1=\angle 3$(Vertically Opposite Angles)
Also, $\angle 2=\angle 5$ and $\angle 1=\angle 6$ (Corresponding Angles)
$\Rightarrow \angle 4=\angle 5$ and $\angle 3=\angle 6$
The converse of the above theorem is also true which states that if the pair of alternate interior angles are equal then the given lines are parallel to each other.

Theorem 2

If two lines which are parallel are intersected by a transversal then the pair of interior angles on the same side of the transversal are supplementary.
∠3+ ∠5=180° and ∠4+∠6=180°
As ∠4=∠5 and ∠3=∠6 (Alternate interior angles)
∠3+ ∠4=180° and ∠5+∠6=180° (Linear pair axiom)
⇒∠3+ ∠5=180° and ∠4+∠6=180°
The converse of the above theorem is also true which states that if the pair of co-interior angles are supplementary then the given lines are parallel to each other.

Solved Examples

Q.1: In the given figure, p || q and l is a transversal. Find the values of x and y.

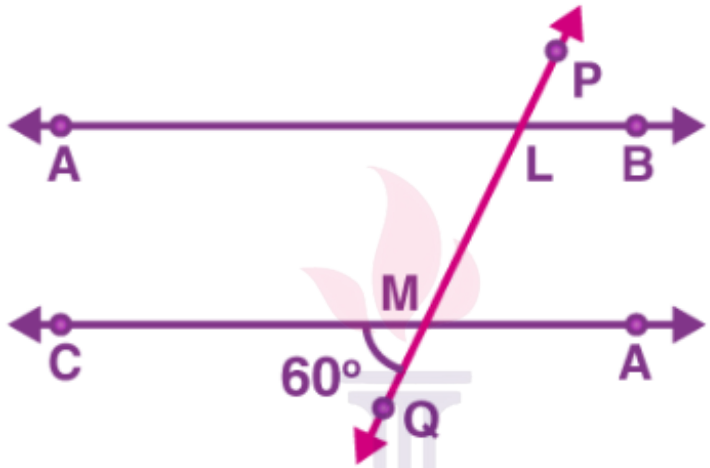


Solution: Since, 6x+y and x+5y are corresponding angles.

6 x + y = x + 5 y
6 x − x = 5 y − y
5 x = 4 y
x = 4y/5 Now, 4x and 6x+y are linear pair of angles, so,
4 x + 6 x + y = 180°10x + y = 180°
40y/5 + y = 180°
45y/5 = 180°
45y = 180 × 5 = 900°
y = 20
x = (4 × 20)/5 = 16
Therefore, x = 16 and y = 20

Solved Examples

Q.2: In Figure, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively, If ∠CMQ = 60, find all other angles in the figure.



Solution:

∠ALM = ∠CMQ = 60° [corresponding angles]

∠LMD = ∠CMQ = 60° [Vertically opposite angles]

∠ALM = ∠PLB = 60 [Vertically opposite angles]

Here, ∠CMQ + ∠QMD = 180° are the linear pair

∠QMD = 180° – 60° = 120°

Now,

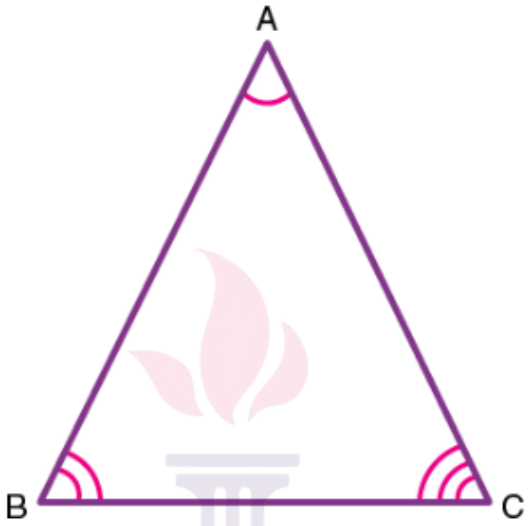
∠QMD = ∠MLB = 120° [Corresponding angles]

∠QMD = ∠CML = 120° [Vertically opposite angles]

∠MLB = ∠ALP = 120° [Vertically opposite angles]

Angle Sum Property of a Triangle Theorem

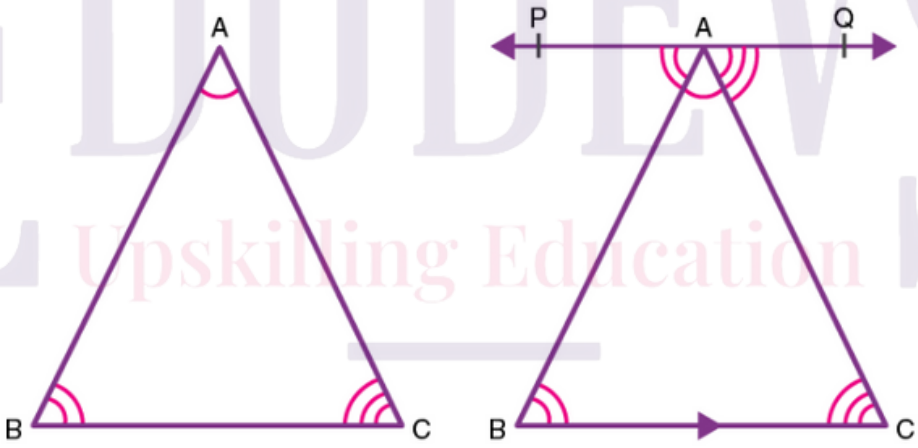
In the given triangle, ΔABC, AB, BC, and CA represent three sides. A, B and C are the three vertices and ∠ABC, ∠BCA and ∠CAB are three interior angles of ΔABC.



Theorem 1: Angle sum property of triangle states that the sum of interior angles of a triangle is 180°.

Proof:

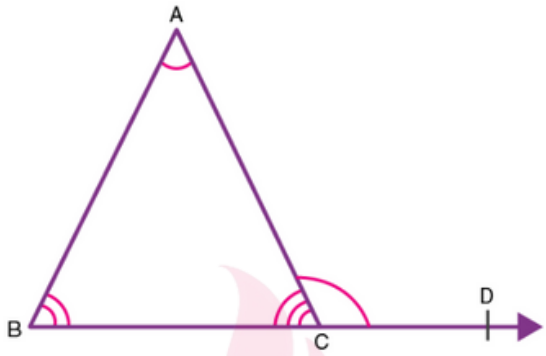
Consider a ΔABC, as shown in the figure below. To prove the above property of triangles, draw a line PQ parallel to the side BC of the given triangle.



Since PQ is a straight line, it can be concluded that:
∠PAB + ∠BAC + ∠QAC = 180° .....(1)
Since PQ||BC and AB, AC are transversals,
Therefore, ∠QAC = ∠ACB (a pair of alternate angle)
Also, ∠PAB = ∠CBA (a pair of alternate angle)
Substituting the value of ∠QAC and∠PAB in equation (1),
∠ACB + ∠BAC + ∠CBA= 180°
Thus, the sum of the interior angles of a triangle is 180°.

Exterior Angle Property of a Triangle Theorem

Theorem 2: If any side of a triangle is extended, then the exterior angle so formed is the sum of the two opposite interior angles of the triangle.



In the given figure, the side BC of ΔABC is extended. The exterior angle ∠ACD so formed is the sum of measures of ∠ABC and ∠CAB.

Proof:

From figure 3, ∠ACB and ∠ACD form a linear pair since they represent the adjacent angles on a straight line.

Thus, ∠ACB + ∠ACD = 180° .....(2)

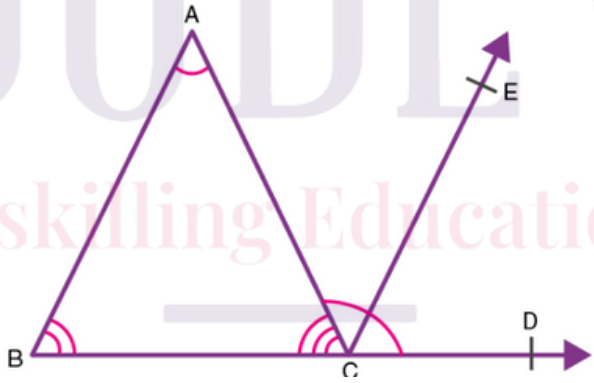
Also, from the angle sum property, it follows that:

∠ACB + ∠BAC + ∠CBA = 180° .....(3)

From equation (2) and (3) it follows that:

∠ACD = ∠BAC + ∠CBA

This property can also be proved using the concept of parallel lines as follows:



In the given figure, side BC of ΔABC is extended. A line CE parallel to the side AB is drawn. Since the line segment BA||CE and AC is the transversal,

∠CAB = ∠ACE .....(4) (Pair of alternate angles)

Also, the line segment BA||CE and BD is the transversal

Therefore, ∠ABC = ∠ECD .....(5) (Corresponding angles)

We have, ∠ACB + ∠BAC + ∠CBA = 180° .....(6)

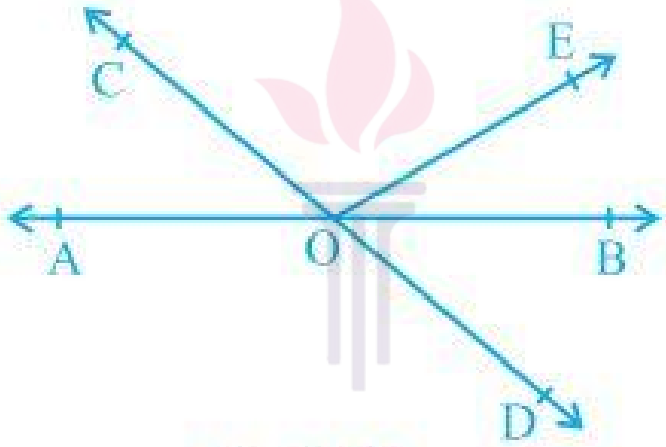
Since the sum of angles on a straight line is 180°

Therefore, ∠ACB + ∠ACE + ∠ECD = 180° .....(7)

Since, ∠ACE + ∠ECD = ∠ACD(From figure 4)

Substituting this value in equation (7);
∠ACB + ∠ACD = 180° ……(8)
From the equations (6) and (8) it follows that,
∠ACD = ∠BAC + ∠CBA
Hence, it can be seen that the exterior angle of a triangle equals the sum of its opposite interior angles.

Q3.In Fig, lines AB and CD intersect at O. If ∠AOC +∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



From the diagram, we have

(∠AOC +∠BOE +∠COE) and (∠COE +∠BOD +∠BOE) forms a straight line.

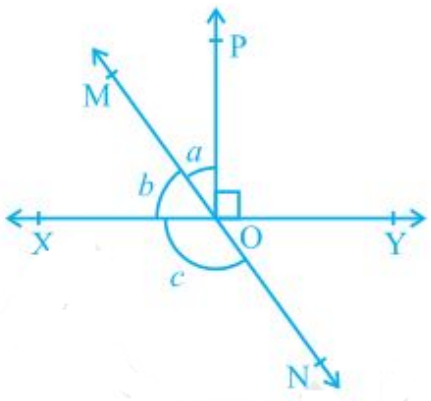
So, ∠AOC+∠BOE +∠COE = ∠COE +∠BOD+∠BOE = 180°

Now, by putting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get

∠COE = 110° and ∠BOE = 30°

So, reflex ∠COE = 360° – 110° = 250°

Q4.In Fig, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.



Solution:

We know that the sum of linear pair is always equal to 180°

So,

∠POY +a +b = 180°

Putting the value of ∠POY = 90° (as given in the question), we get,

a+b = 90°

Now, it is given that a:b = 2:3, so

Let a be 2x and b be 3x

∴ 2x+3x = 90°

Solving this, we get

5x = 90°

So, x = 18°

∴ a = 2×18° = 36°

Similarly, b can be calculated, and the value will be

b = 3×18° = 54°

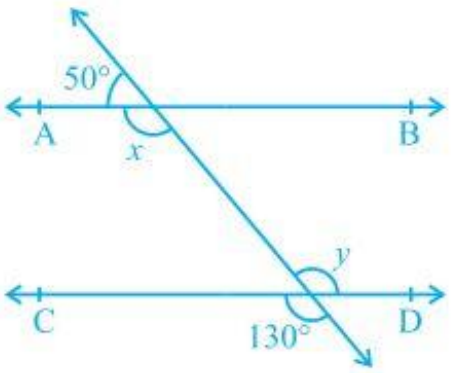
From the diagram, b+c also forms a straight angle, so

b+c = 180°

c+54° = 180°

∴ c = 126°

Q5. In Fig, find the values of x and y and then show that AB || CD.



Solution:

We know that a linear pair is equal to 180°.

So, x+50° = 180°

∴ x = 130°

We also know that vertically opposite angles are equal.

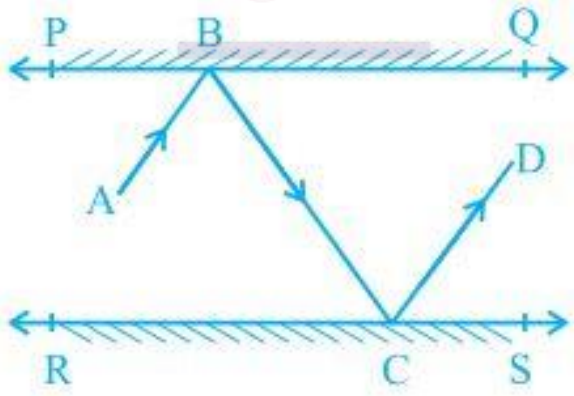
So, y = 130°

In two parallel lines, the alternate interior angles are equal. In this,

x = y = 130°

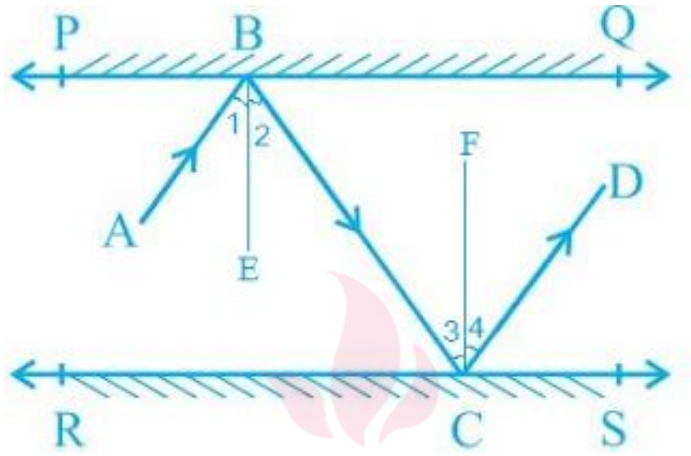
This proves that alternate interior angles are equal, so AB || CD.

Q6. In Fig, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Solution:

First, draw two lines, BE and CF, such that BE ⊥ PQ and CF ⊥ RS.
Now, since PQ || RS,
So, BE || CF



We know that,

Angle of incidence = Angle of reflection (By the law of reflection)

So,

∠1 = ∠2 and

∠3 = ∠4

We also know that alternate interior angles are equal. Here, BE ⊥ CF and the transversal line BC cuts them at B and C

So, ∠2 = ∠3 (As they are alternate interior angles)

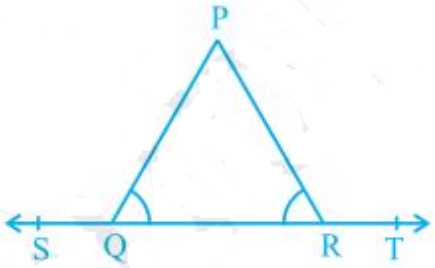
Now, ∠1 +∠2 = ∠3 +∠4

Or, ∠ABC = ∠DCB

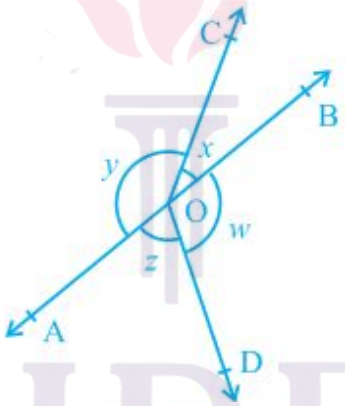
So, AB || CD (alternate interior angles are equal)

ASSIGNMENT FOR CLASS WORK

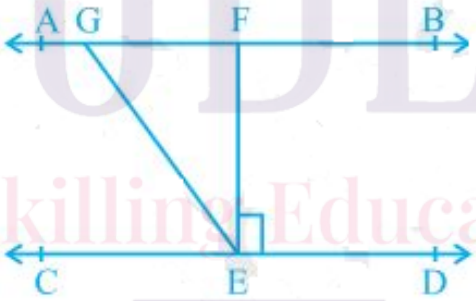
Q1. In Fig, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



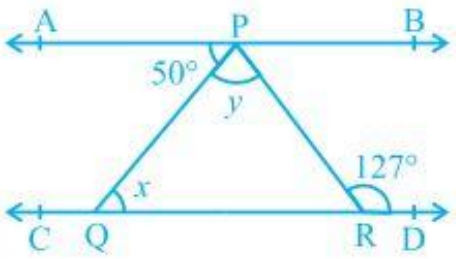
Q2. In Fig, if x+y = w+z, then prove that AOB is a line.



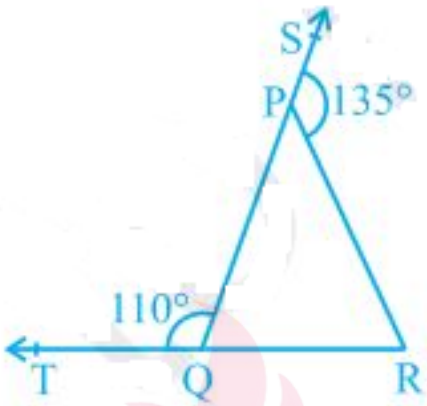
Q3. In Fig, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



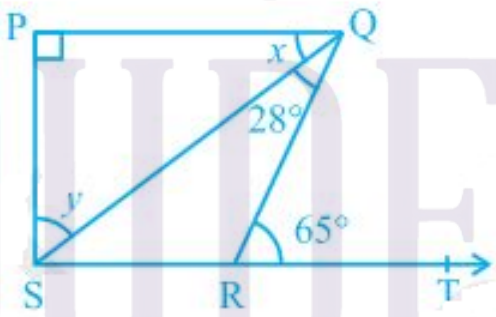
Q4. In Fig, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.



Q5. In Fig, sides QP and RQ of ΔPQR are produced to points S and T,respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.

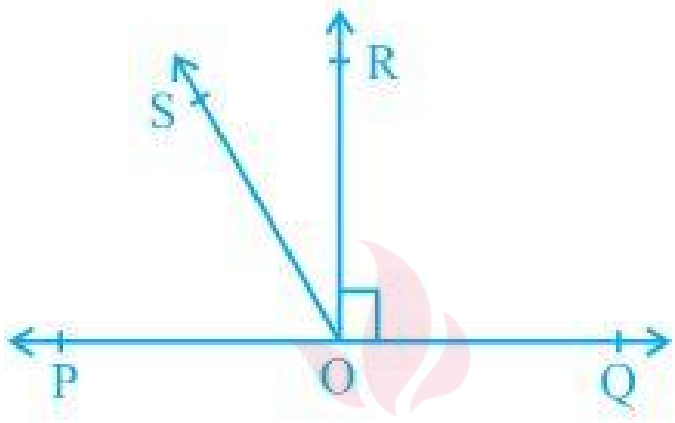


Q6. In Fig,if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of x and y.



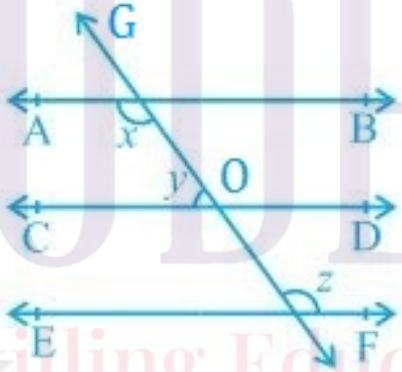
WORKSHEET FOR ASSESSMENT

Q1. In Fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = ½ (∠QOS − ∠POS).

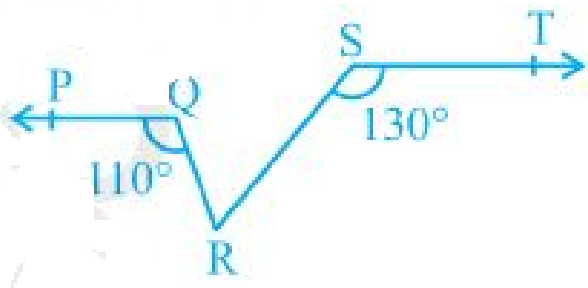


Q2. It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.

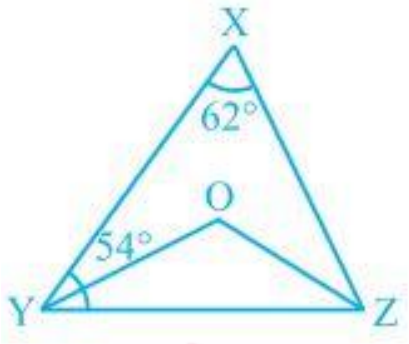
Q3. In Fig, if AB || CD, CD || EF and y : z = 3 : 7, find x.



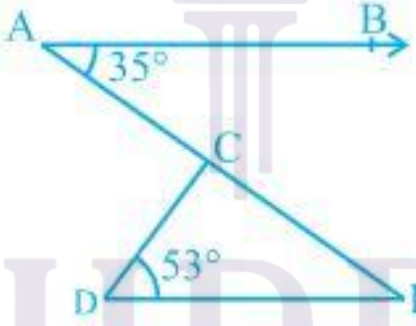
Q4. In Fig, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.
[Hint : Draw a line parallel to ST through point R.]



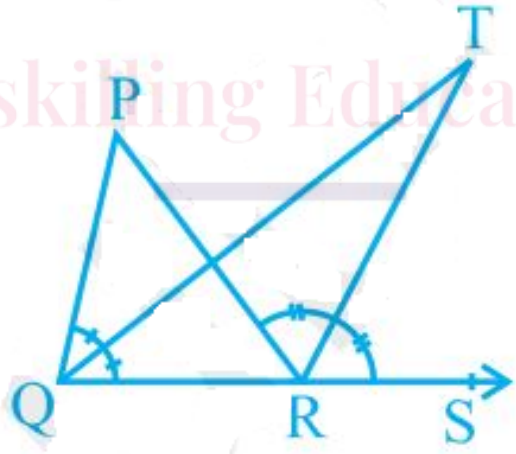
Q5. In Fig, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY, respectively of Δ XYZ, find ∠OZY and ∠YOZ.



Q6. In Fig, if AB || DE, ∠BAC = 35° and ∠CDE = 53°, find ∠DCE.



Q7. In Fig, the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = ½ ∠QPR.



CH 10-HERON'S FORMULA

LESSON PLAN

Class & Section –IX
No of Periods allotted per week: 1
Duration: 40 minutes

Subject: Mathematics
MARCH & APRIL

Chapter – 10 : HERON'S FORMULA

Unit	Chapter 12: Heron's Formula
Class Transaction	Total: 7 periods (approx. 40 min each)
Pre-requisite for the course	This lesson requires 1. Basic knowledge of triangles and quadrilaterals 2. Knowledge of types of triangles. 3. Knowledge of calculating areas of some basictriangles.
Assessment of qualifying knowledge	1. Written test 2. Lab activity 3. Group Discussion 4. HW notebook
Objective	Students would be able to learn to find the area of triangle when the sides of the triangle are given, to find the area of quadrilateral by dividing them into two triangles by using **critical thinking and creative approach** of analyzing the given shapes.
Learning Outcomes	KNOWLEDGE- *Students will **know** and **understand*** 1. The formula for calculating area of an equilateral triangle, right angled triangle. 2. How to use Heron's formula for calculation the area of a triangle whose all the three sides are given. 3. How to calculate the area of a quadrilateral by dividing it into two triangles. SKILLS- *Students would be able to* 1. Find the area of the given triangles easily by using the learnt formulas. 2. Use Heron's formula efficiently and thus would be able to find the area of a triangle whose three sides would be given. 3. Apply Heron's formula in solving day to day real life problems by **critical thinking** and will be able to **analyse** the division of shapes into triangular portions.
Transaction Methodology (The teacher can use the mentioned techniques, wherever applicable, and can use any other too.)	**Strategies Used : Inductive Deductive Reasoning, Graphic Organizers, Think, pair and share, muddiest point discussion** **Dimensions of Learning : D1, D2, D3** Transaction would proceed in the following manner- ***Brain Storming***-The class would start with a discussion on what the students have already learnt in the previous classes and hence what is it that

Sign of Coordinator:

Sign of the Principal:

CHAPTER SUMMARY

What is Heron's Formula?

Heron's formula is a formula to calculate the area of triangles, given the three sides of the triangle. This formula is also used to find the area of the quadrilateral, by dividing the quadrilateral into two triangles, along its diagonal.
If a, b and c are the three sides of a triangle, respectively, then Heron's formula is given by:

$Area\ of\ triangle\ using\ three\ sides = \sqrt{s(s-a)(s-b)(s-c)}$

Semiperimeter, s = Perimeter of triangle/2 = (a+b+c)/2

Heron's Formula for Triangles

According to Heron, we can find the area of any given triangle, whether it is a scalene, isosceles or equilateral, by using the formula, provided the sides of the triangle.
Suppose, a triangle ABC, whose sides are a, b and c, respectively. Thus, the area of a triangle can be given by;

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where "s" is semi-perimeter = (a+b+c) / 2
And a, b, c are the three sides of the triangle.

Proof of Heron's Formula

There are two methods by which we can derive Heron's formula.

- First, by using trigonometric identities and cosine rule.
- Secondly, solving algebraic expressions using the Pythagoras theorem.

Let us see one by one both the proofs or derivation.

Using Cosine Rule

Let us prove the result using the law of cosines:
Let a, b, c be the sides of the triangle and α, β, γ are opposite angles to the sides.
We know that, the law of cosines is

$$cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

Again, using trig identity, we have

$$sin\gamma = \sqrt{1 - cos^2\gamma}$$

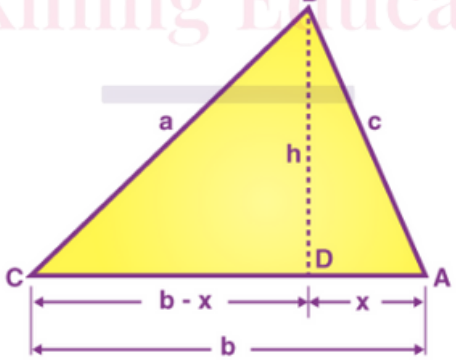
$$= \frac{\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}}{2ab}$$

Here, Base of triangle = a
Altitude = b sinγ
Now,

$$\begin{aligned}
A &= \frac{1}{2}(\text{base})(\text{altitude}) \\
&= \frac{1}{2}ab\sin\gamma \\
&= \frac{1}{4}\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2} \\
&= \frac{1}{4}\sqrt{(2ab - (a^2+b^2-c^2))(2ab + (a^2+b^2-c^2))} \\
&= \frac{1}{4}\sqrt{(c^2 - (a-b)^2)((a+b)^2 - c^2)} \\
&= \sqrt{\frac{(c-(a-b))(c+(a-b))((a+b)-c)((a+b)+c)}{16}} \\
&= \sqrt{\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}\frac{(a+b+c)}{2}} \\
&= \sqrt{\frac{(a+b+c)}{2}\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\frac{(a+b-c)}{2}} \\
&= \sqrt{s(s-a)(s-b)(s-c)}.
\end{aligned}$$

Using Pythagoras Theorem

Area of a Triangle with 3 Sides
Area of ΔABC is given by



A = 1/2 bh _ _ _ _ (i)
Draw a perpendicular BD on AC
Consider a ΔADB
x2 + h2 = c2
x2 = c2 − h2—(ii)
⇒ x = √(c2−h2)−−−−−−−(iii)
Consider a ΔCDB,
(b−x)2 + h2 = a2
(b−x)2 = a2 − h2
b2 − 2bx + x2 = a2−h2
Substituting the value of x and x2 from equation (ii) and (iii), we get
b2 − 2b√(c2−h2)+ c2−h2 = a2 − h2
b2 + c2 − a2 = 2b√(c2 − h2)
Squaring on both sides, we get;
(b2+c2−a2)2 = 4b2(c2−h2)

$$\frac{(b^2+c^2-a^2)^2}{4b^2} = c^2-h^2$$

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{4b^2c^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2-(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{[2bc+(b^2+c^2-a^2)][2bc-(b^2+c^2-a^2)]}{4b^2}$$

$$h^2 = \frac{[(b^2+2bc+c^2)-a^2][a^2-(b^2-2bc+c^2)]}{4b^2}$$

$$h^2 = \frac{[(b+c)^2-a^2].[a^2-(b-c)^2]}{4b^2}$$

$$h^2 = \frac{[(b+c)+a][(b+c)-a].[a+(b-c)][a-(b-c)]}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

The perimeter of a ΔABC is given by:
P = a + b + c
Thus, substituting a + b + c = P in the above equation, we get;

$$\Rightarrow h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

$$\Rightarrow h = \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

Substituting the value of h in equation (i), we get;

$$A = \frac{1}{2} b \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$A = \frac{1}{4} \sqrt{(P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{1}{16} P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{P}{2}\left(\frac{P-2a}{2}\right)\left(\frac{P-2b}{2}\right)\left(\frac{P-2c}{2}\right)}$$

$$Semi\ perimeter(s) = \frac{perimeter}{2} = \frac{P}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Note: Heron's formula is applicable to all types of triangles and the formula can also be derived using the law of cosines and the law of Cotangents.

Heron's Formula for Equilateral Triangle

As we know the equilateral triangle has all its sides equal. To find the area of the equilateral triangle let us first find the semi perimeter of the equilateral triangle will be:
s = (a + a + a)/2
s = 3a/2
where a is the length of the side.
Now, as per the heron's formula, we know;

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Since, a = b = c
Therefore,
A = √[s(s − a)3]
which is the required formula.

Heron's Formula For Quadrilateral

Let us learn how to find the area of quadrilateral using Heron's formula here.

If ABCD is a quadrilateral, where AB||CD and AC & BD are the diagonals.

AC divides the quad.ABCD into two triangles ADC and ABC.

Now we have two triangles here.

Area of quad.ABCD = Area of ΔADC + Area of ΔABC

So, if we know the lengths of all sides of a quadrilateral and the length of diagonal AC, then we can use Heron's formula to find the total area.

Hence, we will first find the area of ΔADC and area of ΔABC using Heron's formula and at last, will add them to get the final value.

Solved Examples

1. Find the area of a trapezium, the length of whose parallel sides is given as 22 cm and 12 cm and the length of other sides is 14 cm each.

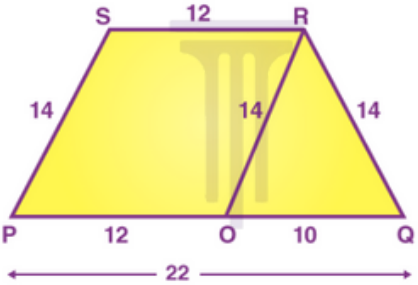
Solution: Let PQRS be the given trapezium in which PQ = 22 cm, SR = 12 cm,
PS = QR = 14 cm.

Constructions: Draw OR||PS

Now, PORS is a parallelogram in which PS||OR and PO||SR

Therefore, PO = SR = 12 cm

⇒ OQ = PQ − PO = 22 − 12 = 10 cm



In ΔOQR , we have

In ΔOQR , we have

$s = \frac{14+14+10}{2} = \frac{38}{2} = 19$

$\text{Area of } \Delta OQR = \sqrt{s(s-a)(s-b)(s-C)}$

$= \sqrt{(19(19-14)(19-14)(19-10))}$

$= \sqrt{4275}$

$= 15\sqrt{19}cm^2 \ldots\ldots\ldots (i)$

$\text{We know that Area} = \frac{1}{2} \times b \times h$

$\Rightarrow 15\sqrt{19} = \frac{1}{2} \times 10 \times h$

$\Rightarrow h = 5\sqrt{19} \ldots\ldots\ldots\ldots\ldots (ii)$

$\text{Area of trapezium} = \frac{1}{2}\ (PQ + SR) \times h$

$= \frac{1}{2}(22 + 12) \times 3\sqrt{19}$

$= 51\sqrt{19}cm^2$

2. Find the area of the triangle whose sides measure 10 cm, 17 cm and 21 cm. Also, determine the length of the altitude on the side which measures 17 cm.

Solution:

$$s = \frac{a+b+c}{2} = \frac{10+17+21}{2} = 24$$

$$\text{Area of Triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\sqrt{24 \times 14 \times 7 \times 3}$$

$$\sqrt{7056} = 84 \; square \; cm$$

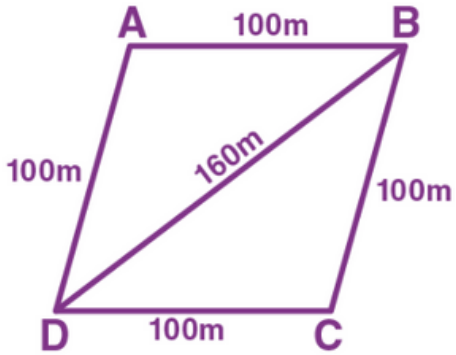
Taking 17 cm as the base length we need to find the height
Area, A = 1/2 × base × height
1/2 × 17 × h = 84 or h = 168/17 = 9.88 cm (Rounded to the nearest hundredth).

3. A triangle PQR has sides 4 cm, 13 cm and 15 cm. Find the area of the triangle.

Solution:
Semiperimeter of triangle PQR, s = (4 + 13 + 15)/2 = 32/2 = 16
By heron's formula, we know;
A = √[s(s − a)(s − b)(s − c)]
Hence, A = √[16(16 − 4)(16 − 13)(16 − 15)] = √(16 × 12 × 3 × 1) = √576 = 24 sq.cm

4. Masha has a piece of land which is shaped like a rhombus. She wants her one son and one daughter to work on the land to produce different crops. So, she has divided the land into two equal parts. Determine how much area each of them will get for their crop production if the perimeter of the land is 400 m and one of its diagonals is 160 m.

Solution:
Given: ABCD is a rhombus-shaped land.



Given that, perimeter = 400 m

Therefore, each side length of rhombus = 400/4 = 100 cm.

Thus, we can take that AD = AB = 100 m

Diagonal, BD = 160 m.

Now, consider the triangle ABD,

The semiperimeter of Δ ABD, s = (100+ 100+ 160)/2

s = 360/2 m

s = 180 m.

Therefore, the semi perimeter of a triangle = 180m.

Thus, the area of triangle ABD = √[s(s-a)(s-b)(s-c)] square units.

Now, substitute s= 180 m, a= 100, b= 100 and c= 160 in the Heron's formula, we get

A = √[180 (180 – 100)(180 – 100)(180 – 160)] m2

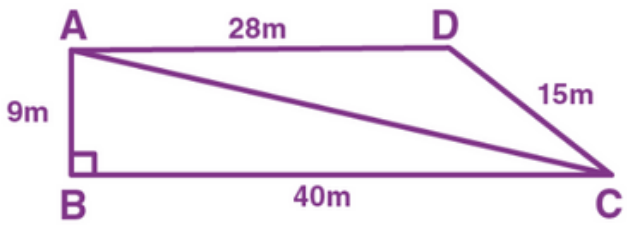
A = √[180(80)(80)(20)] m2

A = √23040000 m2

A = 4800 m2

Hence, each of them will get land for their crop production of 4800 m2.

5. The school students staged a rally for a cleanliness campaign. They walked through the lane into two groups. One group walked through the lanes AB, BC and CA, and the other group walked through the lanes AC, CD and DA as shown in the figure. They cleaned the area enclosed within the lanes. Find the total area cleaned by the students (Neglect the width of the lane), if AB = 9m, BC = 40m, CD = 15m, DA = 28m and ∠B = 90°. Also, find which group cleaned more area and how much.



Solution:

Given that, AB = 9m, BC = 40m, and ∠B = 90º

Therefore, AC = √(92+402) = √(81+1600)

AC = √1681 = 41m.

Thus, the first group of students cleaned the area of triangle ABC (which is right-angled)

Therefore, the area of triangle = (½)(40)(9) m2

A = 180m2

Hence, the first group of students to clean the lane ABC is 180m2.

Now, the second group of students clean the lane ACD, which is a scalene triangle having side lengths 41m, 28m and 15m.

Now, use Heron's formula to find the area of ACD.

So, s = (41+28+15)/2

s = 42m

By, using Heron's formula, A =√[s(s-a)(s-b)(s-c)], we get

A = √[42(42-41)(42-28)(42-15)] m2

A = √[42(1)(14)(27)] m2

A = √15876 m2

A = 126 m2

Hence, the second group of students cleaned the lane ACD is 126m2.
The total area cleaned by all the students = (180+126)m2 = 306m2.
Therefore, the first group of students cleaned more lane areas than the area cleaned by the second group of students.

Assignment for Class Work

1.Find the area of a triangle whose perimeter is 54 cm and two of its sides measure 12 cm and 25 cm.

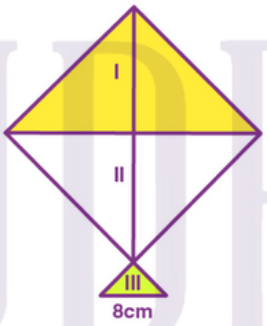
2.If the length of equal sides of an isosceles triangle is 5 cm and base is 6 cm, then find its area using heron's formula.

3.The sides of a quadrilateral field, taken in order are 26 cm, 27 cm, 7 cm, 24 cm respectively. The angle contained by the last two sides is a right angle. Find its area.

3.The sides of a quadrilateral field, taken in order are 26 cm, 27 cm, 7 cm, 24 cm respectively. The angle contained by the last two sides is a right angle. Find its area.

4.A field is in the shape of a rhombus and it has green grass for 18 cows to graze. Determine how much area of grass field each cow will be getting, if each side of the rhombus is 30m and its longest diagonal is 48m.

5.A kite is in a square shape. Its diagonal is 32 cm and its isosceles triangle of base 8 cm and side 6 cm each is made of three different shades as shown in the figure. Find how much paper of each shade has been used in it to make a kite.



6.Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42cm.

7.Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42cm.

8.A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

9.A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

WORKSHEET FOR ASSESSMENT

1.Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

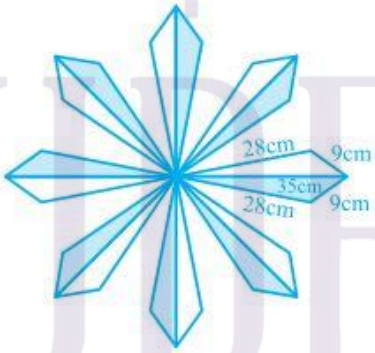
2.An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

3.A park, in the shape of a quadrilateral ABCD, has C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

4.Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

4.A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

5.A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (in fig). Find the cost of polishing the tiles at the rate of 50p per cm2 .



6.The area of triangle with given two sides 18cm and 10cm respectively and perimeter equal to 42 cm is

7.The base of an isosceles right triangle is 30 cm. The area of triangle is:

8.The perimeter of a triangular field is 540 m and its sides are in the ratio 25:17:12. Find the area of the triangle.

9. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

CH 11-SURFACE AREAS AND VOLUMES

LESSON PLAN

Class & Section –IX
No of Periods allotted per week: 1
Duration: 40 minutes

Subject: Mathematics
MARCH & APRIL

Chapter – 11 : Surface Areas And Volumes

Unit	Surface Areas and Volumes (Chapter 13: Surface Areas And Volumes)
Class Transaction	Total: 15 periods (approx. 40 min each)
Pre-requisite for the course	This lesson requires 1. Basic knowledge of triangles and circles. 2. Knowledge of square, rectangle, parallelograms and solid figures. 3. Knowledge of calculating perimeter and area of square and rectangles.
Assessment of qualifying knowledge	1. Written test 2. Lab activity 3. HW notebook
Objective	Students would be able to find • surface area of cube and cuboid, • surface area of a right circular cylinder, • surface area of a right circular cone, • surface area of a sphere, • volume of cuboid, • volume of cylinder, • volume of right circular cone, • volume of sphere, • volume of hemisphere, thereby inculcating competencies like **collaboration, critical thinking and creativity**
Learning Outcomes	KNOWLEDGE- *Students will **develop** the ability to**understand and find*** 1. Total surface area and curved surface area of solid shapes. 2. Volume of solid shapes. SKILLS and COMPETENCIES-*Students would be able to* 1. Find the area and perimeter of base of square, rectangle, parallelogram, triangle and circle. 2. Solve questions based on the topics like area and volume of solid figures. 3. Use analytical skills to visualize the given scenario and use the concepts learnt in everyday problems. 4. Use synthetic skills to solve problems.
Transaction Methodology (The teacher can use the mentioned techniques, wherever applicable, and can use any other too.)	Strategies Used- **Graphic Organizers, Inductive deductive reasoning, think pair share, Muddiest point discussion** Dimensions of learning- **D1, D2, D3** Transaction would proceed in the following manner- ***Brain Storming***-The class would start with a discussion on what the students have already learnt in the previous classes and hence what is it that they would learn now. They would also be told the significance of the topic that they would be studying.

Sign of Coordinator:

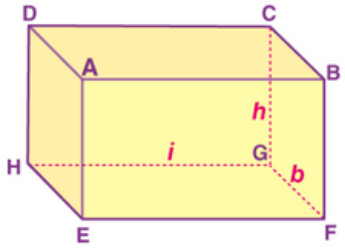
Sign of the Principal:

CHAPTER SUMMARY

Cuboid

A cuboid is a three-dimensional Shape. The cuboid is made of six rectangular faces, which are placed at right angles. The total surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Total Surface Area of a Cuboid



Consider a cuboid whose length is "l" cm, breadth is b cm and height h cm.
Area of face ABCD = Area of Face EFGH = (l × b) cm2
Area of face AEHD = Area of face BFGC = (b × h) cm2
Area of face ABFE = Area of face DHGC = (l × h) cm2
Total surface area (TSA) of cuboid = Sum of the areas of all its six faces
= 2(l × b) + 2(b × h) + 2(l × h)

- TSA (cuboid)= 2(lb + bh +lh)

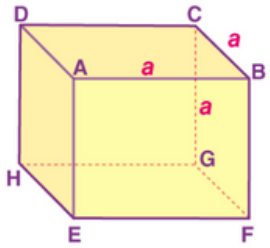
Lateral Surface Area of a Cuboid

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
The lateral surface area of the cuboid
= Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC
= 2(b × h) + 2(l × h)
LSA (cuboid) = 2h(l + b)

Cube

A cuboid whose length, breadth and height are all equal is called a cube. It is a three-dimensional shape bounded by six equal squares. It has 12 edges and 8 vertices.

Total Surface Area of a Cube



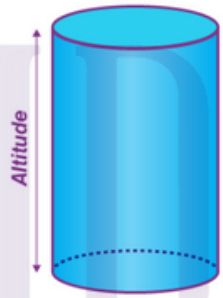
For cube, length = breadth = height
Suppose the length of an edge = a
Total surface area(TSA) of the cube = 2(a × a + a × a + a × a)
TSA(cube) = 2 × (3a2) = 6a2

Lateral Surface Area of a Cube

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
Lateral surface area of cube = 2(a × a + a × a) = 4a2

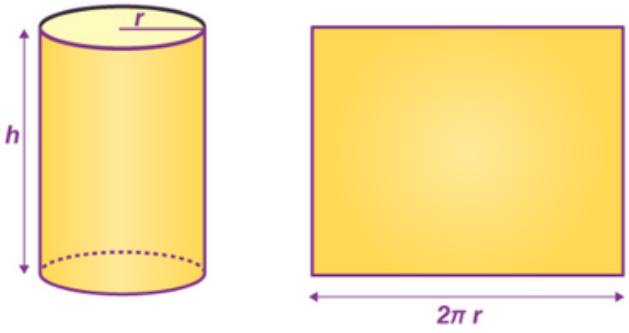
Right Circular Cylinder

A right circular cylinder is a closed solid that has two parallel circular bases connected by a curved surface in which the two bases are exactly over each other and the axis is at right angles to the base.



Curved Surface Area of a Right Circular Cylinder

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter (d = 2r) of the circular base, will be transformed into a rectangle of length 2πr and height h units. Thus,



Curved surface area(CSA) of a cylinder of base radius r and height h = 2π × r × h

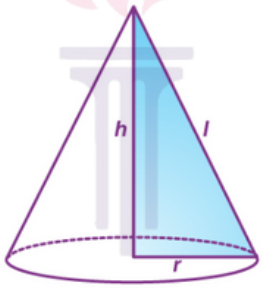
Total Surface Area of a Right Circular Cylinder

Total surface area(TSA) of a cylinder of base radius r and height h = 2π × r × h + area of two circular bases
⇒ TSA = 2π × r × h + 2 × πr2
⇒ TSA = 2πr(h + r)

Right Circular Cone

A right circular cone is a circular cone whose axis is perpendicular to its base.

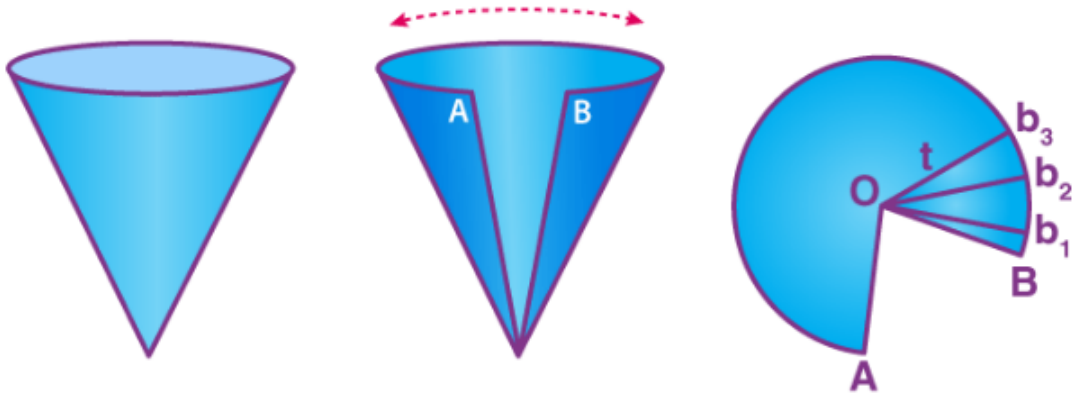
Relation between Slant Height and Height of a Right Circular Cone



he relationship between slant height(l) and height(h) of a right circular cone is:
l2 = h2 + r2 (Using Pythagoras Theorem)
Where r is the radius of the base of the cone.

Curved Surface Area of a Right Circular Cone

Consider a right circular cone with slant length l and radius r.
If a perpendicular cut is made from a point on the circumference of the base to the vertex and the cone is opened up, a sector of a circle with radius l is produced, as shown in the figure below:



Label A and B and corresponding b1, b2 …bn at equal intervals, with O as the common vertex. The Curved surface area(CSA) of the cone will be the sum of the areas of the small triangles: 1/2 × (b1 + b2.……..bn) × l
(b1 + b2...bn) is also equal to the circumference of base = 2πr
CSA of right circular cone = (1/2) × (2πr) × l = πrl (On substituting the values)

Total Surface Area of a Right Circular Cone

Total surface area(TSA) = Curved surface area(CSA) + area of base = πrl + πr2 = πr(l + r)

Sphere

A sphere is a closed three-dimensional solid figure, where all the points on the surface of the sphere are equidistant from the common fixed point called “centre”. The equidistant is called the “radius”.

Surface Area of a Sphere

The surface area of a sphere of radius r = 4 times the area of a circle of radius r =4×(πr2)
For a sphere Curved surface area (CSA) = Total Surface area(TSA) = 4πr2

Shapes	Surface Areas
Cuboid	$2(lb + bh + hl)$
Cube	$6a^2$
Right Circular Cylinder	$2\pi r(r + h)$
Right Circular Cone	$\pi r(l + r)$, $(l^2 = h^2 + r^2)$
Sphere	$4\pi r^2$

Volume of a Cuboid

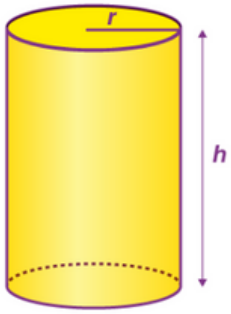
The volume of a cuboid is the product of its dimensions.
Volume of a cuboid = length × breadth × height= lbh
Where l is the length of the cuboid, b is the breadth, and h is the height of the cuboid

Volume of a Cube

The volume of a cube = base area × height.
Since all dimensions are identical, the volume of the cube = a3
Where a is the length of the edge of the cube.

Volume of a Right Circular Cylinder

The volume of a right circular cylinder is equal to base area × its height.



The volume of a cylinder =πr2h
Where r is the radius of the base of the cylinder and h is the height of the cylinder.

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times the volume of a cylinder with the same radius and height. In other words, three cones make one cylinder of the same height and base.
The volume of right circular cone = (1/3)πr2h
Where r is the radius of the base of the cone and h is the height of the cone.

Volume of a Sphere

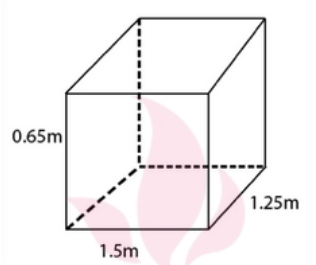
The volume of a sphere of radius r =(4/3)πr3

Shapes	Volumes
Cuboid	length × breadth × height
Cube	a^3
Right Circular Cylinder	$\pi r^2 h$
Right Circular Cone	$\frac{1}{3} \pi r^2 h$
Sphere	$4/3\ \pi\ r^3$

Solved Examples

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine
(I) The area of the sheet required for making the box.
(ii) The cost of the sheet for it, if a sheet measuring 1m2 costs Rs. 20.

Sol:



Given: length (l) of box = 1.5m
Breadth (b) of box = 1.25 m
Depth (h) of box = 0.65m
(i) Box is to be open at the top.
Area of sheet required.
= 2lh+2bh+lb
= [2×1.5×0.65+2×1.25×0.65+1.5×1.25]m2
= (1.95+1.625+1.875) m2 = 5.45 m2
(ii) Cost of sheet per m2 area = Rs.20
Cost of sheet of 5.45 m2 area = Rs (5.45×20)
= Rs.109.

2. The length, breadth and height of a room are 5 m, 4 m and 3 m, respectively. Find the cost of whitewashing the walls of the room and ceiling at the rate of Rs 7.50 per m2.

Solution:
Length (l) of room = 5m
Breadth (b) of room = 4m
Height (h) of room = 3m
It can be observed that four walls and the ceiling of the room are to be whitewashed.
Total area to be whitewashed = Area of walls + Area of the ceiling of the room
= 2lh+2bh+lb
= [2×5×3+2×4×3+5×4]
= (30+24+20)
= 74
Area = 74 m2
Also,
Cost of whitewash per m2 area = Rs.7.50 (Given)
Cost of whitewashing 74 m2 area = Rs. (74×7.50)
= Rs. 555

Solved Examples

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m2 is Rs.15,000, find the height of the hall.
[Hint: Area of the four walls = Lateral surface area.]

Sol:

Let the length, breadth, and height of the rectangular hall be l, b, and h, respectively.
Area of four walls = 2lh+2bh
= 2(l+b)h
Perimeter of the floor of hall = 2(l+b)
= 250 m
Area of four walls = 2(l+b) h = 250h m2
Cost of painting per square metre area = Rs.10
Cost of painting 250h square metre area = Rs (250h×10) = Rs.2500h
However, it is given that the cost of painting the walls is Rs. 15,000.
15000 = 2500h
Or h = 6
Therefore, the height of the hall is 6 m.

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m2. How many bricks of dimensions 22.5 cm×10 cm×7.5 cm can be painted out of this container?

Solution:
Total surface area of one brick = 2(lb +bh+lb)
= [2(22.5×10+10×7.5+22.5×7.5)] cm2
= 2(225+75+168.75) cm2
= (2×468.75) cm2
= 937.5 cm2
Let n bricks can be painted out by the paint of the container.
Area of n bricks = (n×937.5) cm2 = 937.5n cm2
As per the given instructions, the area that can be painted by the paint of the container = 9.375 m2 = 93750 cm2
So, we have 93750 = 937.5n
n = 100
Therefore, 100 bricks can be painted out by the paint of the container.

5. Praveen wanted to make a temporary shelter for her car by making a box-like structure with a tarpaulin that covers all four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how many tarpaulins would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?

Solution:
Let l, b and h be the length, breadth and height of the shelter.
Given:
l = 4m
b = 3m
h = 2.5m
Tarpaulins will be required for the top and four wall sides of the shelter.
Using formula, area of tarpaulin required = 2(lh+bh)+lb
On putting the values of l, b and h, we get
= [2(4×2.5+3×2.5)+4×3] m2
= [2(10+7.5)+12]m2
= 47m2
Therefore, 47 m2 of tarpaulin will be required.

6. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square metres of the sheet are required for the same? Assume π = 22/7

Solution:
Let h be the height and r be the radius of a cylindrical tank.
Height of cylindrical tank, h = 1m
Radius = half of diameter = (140/2) cm = 70cm = 0.7m
Area of sheet required = Total surface area of tank = 2πr(r+h) unit square
= [2×(22/7)×0.7(0.7+1)]
= 7.48 square metres
Therefore, 7.48 square metres of the sheet are required.

7. A metal pipe is 77 cm long. The inner diameter of a cross-section is 4 cm, the outer diameter being 4.4cm. Find its
(i) inner curved surface area
(ii) outer curved surface area
(iii) total surface area
(Assume π=22/7)

Solution:
Let r1 and r2 inner and outer radii of the cylindrical pipe.
r1 = 4/2 cm = 2 cm
r2 = 4.4/2 cm = 2.2 cm
Height of cylindrical pipe, h = length of cylindrical pipe = 77 cm
(i) Curved surface area of the outer surface of pipe = 2πr1h
= 2×(22/7)×2×77 cm2
= 968 cm2

(ii) Curved surface area of the outer surface of pipe = 2πr2h
= 2×(22/7)×2.2×77 cm2
= (22×22×2.2) cm2
= 1064.8 cm2
(iii) Total surface area of pipe = inner curved surface area+ outer curved surface area+ area of both circular ends of pipe
= 2r1h+2r2h+2π(r12-r22)
= 9668+1064.8+2×(22/7)×(2.22-22)
= 2031.8+5.28
= 2038.08 cm2
Therefore, the total surface area of the cylindrical pipe is 2038.08 cm2.
4. The diameter of a roller is 84 cm, and its length is 120 cm. It takes 500 complete revolutions to
move once over to level a playground. Find the area of the playground in m2 (Assume π = 22/7).
Solution:
A roller is shaped like a cylinder.
Let h be the height of the roller and r be the radius.
h = Length of roller = 120 cm
Radius of the circular end of roller = r = (84/2) cm = 42 cm
Now, CSA of roller = 2πrh
= 2×(22/7)×42×120
= 31680 cm2
Area of field = 500×CSA of roller
= (500×31680) cm2
= 15840000 cm2
= 1584 m2.
Therefore, the area of the playground is 1584 m2.

8. Find
(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5m high.
(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank? (Assume π = 22/7)

Solution:
Height of cylindrical tank, h = 4.5m
Radius of the circular end , r = (4.2/2)m = 2.1m
(i) The lateral or curved surface area of the cylindrical tank is 2πrh.
= 2×(22/7)×2.1×4.5 m2
= (44×0.3×4.5) m2
= 59.4 m2
Therefore, the CSA of the tank is 59.4 m2.
(ii) Total surface area of tank = 2πr(r+h)
= 2×(22/7)×2.1×(2.1+4.5)
= 44×0.3×6.6
= 87.12 m2Now, Let S m2 steel sheet be actually used in making the tank.
S(1 -1/12) = 87.12 m2
This implies, S = 95.04 m2
Therefore, 95.04m2 steel was used in actuality while making such a tank.

9. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Solution:
Given:
Length of the cuboidal vessel, l = 10 m
Width of the cuboidal vessel, b = 8m
Volume of the cuboidal vessel, V = 380 m3
Let the height of the given vessel be h.
Formula for volume of a cuboid, V = l×b×h
Using the formula, we have
l×b×h = 380
10×8×h= 380
Or h = 4.75
Therefore, the height of the vessels is 4.75 m.

10. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m3.

Solution:
The given pit has its length(l) as 8m, width (b)as 6m and depth (h)as 3 m.
Volume of cuboidal pit = l×b×h = (8×6×3) = 144 (using formula)
The required Volume is 144 m3
Now,
Cost of digging per m3 volume = Rs 30
Cost of digging 144 m3 volume = Rs (144×30) = Rs 4320

11. A village, having a population of 4000, requires 150 litres of water per head per day.
It has a tank measuring 20 m×15 m×6 m. For how many days will the water in this tank last?

Solution:
Length of the tank = l = 20 m
Breadth of the tank = b = 15 m
Height of the tank = h = 6 m
Total population of a village = 4000
Consumption of water per head per day = 150 litres
Water consumed by the people in 1 day = (4000×150) litres = 600000 litres …(1)
Formula to find the capacity of the tank, C = l×b×h
Using the given data, we have
C = (20×15×6) m3= 1800 m3
Or C = 1800000 litres
Let water in this tank last for d days.
Water consumed by all people in d days = Capacity of the tank (using equation (1))
600000 d =1800000
d = 3
Therefore, the water in this tank will last for 3 days.

Assignment for Class Work

1. A cubical box has each edge 10 cm, and another cuboidal box is 12.5cm long, 10 cm wide, and 8 cm high.
(i) Which box has the greater lateral surface area, and by how much?
(ii) Which box has the smaller total surface area, and by how much?

2. A small indoor greenhouse (herbarium) is made entirely of glass panes (including the base) held together with tape. It is 30cm long, 25 cm wide, and 25 cm high.
(i) What is the area of the glass?
(ii)How much tape is needed for all 12 edges?

3. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm, and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm2, find the cost of cardboard required for supplying 250 boxes of each kind.

3. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm, and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm2, find the cost of cardboard required for supplying 250 boxes of each kind.

4. The curved surface area of a right circular cylinder of height 14 cm is 88 cm2. Find the diameter of the base of the cylinder (Assume π =22/7).

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m2.
(Assume π = 22/7)

6.Curved surface area of a right circular cylinder is 4.4 m2. If the radius of the base of the cylinder is 0.7 m, find its height. (Assume π = 22/7)

7.The inner diameter of a circular well is 3.5m. It is 10m deep. Find
(i) its inner curved surface area.
(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m2.
(Assume π = 22/7)

Assignment for Class Work

8.In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. (Assume π = 22/7)

9. The students of Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? (Assume π =22/7)

10. Diameter of the base of a cone is 10.5 cm, and its slant height is 10 cm. Find its curved surface area. (Assume π=22/7)

11.Find the total surface area of a cone, if its slant height is 21 m and the diameter of its base is 24 m. (Assume π = 22/7)

12.Curved surface area of a cone is 308 cm2, and its slant height is 14 cm. Find
(i) radius of the base and (ii) total surface area of the cone.
(Assume π = 22/7)

13.A conical tent is 10 m high, and the radius of its base is 24 m. Find
(i) slant height of the tent.
(ii) cost of the canvas required to make the tent, if the cost of 1 m2 canvas is Rs 70.
(Assume π=22/7)

14.What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14]

15.The slant height and base diameter of the conical tomb are 25m and 14 m, respectively. Find the cost of whitewashing its curved surface at the rate of Rs. 210 per 100 m2. (Assume π = 22/7)

16.A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates, each measuring 1.5m×1.25 m×0.5 m, that can be stored in the godown.

17.A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

WORKSHEET FOR ASSESSMENT

1. **A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume π =22/7)**

2. Find the surface area of a sphere of radius
(i) 10.5cm (ii) 5.6cm (iii) 14cm
(Assume π=22/7)

3. Find the surface area of a sphere of diameter
(i) 14cm (ii) 21cm (iii) 3.5cm
(Assume π = 22/7)

4. Find the total surface area of a hemisphere of radius 10 cm. [Use π=3.14]

5. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

6. A hemispherical bowl made of brass has an inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm2. (Assume π = 22/7)

7. Find the radius of a sphere whose surface area is 154 cm2. (Assume π = 22/7)

8. A right circular cone is 5.4cm high and radius of its base is 2cm. It is melted and recast into another right circular cone with radius of base as 1.5 cm. Find the height of the new cone formed

9. If the volume of a sphere is divided by its surface area then the result is 27. Find the radius of sphere

10. The curved surface area and volume of a cylindrical pillar are 264cm3 and 396cm3.. Find the diameter and height of the pillar

11. The paint in a certain container is sufficient to paint an area equal to 9.375 sqm. How many bricks of dimensions 22.5cm x 10cm x 7.5cm container can be painted out of this?

12. A solid cube of side 12 cm is cut into 8 cubes of equal volumes. Find the side of new cube

13. The total surface area of a solid hemisphere is 1848cm2 . Find the volume of the hemisphere.

14. The sum of the radius of the base and height of a cylinder is 37m. If the total surface area of the Solid cylinder is 1628m2 , find the volume of cylinder

15. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m3= 1000 l)

WORKSHEET FOR ASSESSMENT

16. The volume of a right circular cone is 9856cm3. If the diameter of the base is 28cm, find
(i) height of the cone
(ii) slant height of the cone
(iii) curved surface area of the cone
(Assume π = 22/7)

17. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

18. Find the amount of water displaced by a solid spherical ball of diameter
(i) 28 cm (ii) 0.21 m
(Assume π =22/7)

19.The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm3? (Assume π=22/7)ases.

20.The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

21.A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm3) is needed to fill this capsule? (Assume π = 22/7)

22.Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the
(i) radius r' of the new sphere,
(ii) ratio of Sand S'.

23.A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 4989.60. If the cost of white-washing is 20 per square metre, find the
(i) inside surface area of the dome (ii) volume of the air inside the dome
(Assume π = 22/7)

24. The Volume of a cylinder is 448 π cm3 and height 7 cm. Find its total surface area.

25. Two cubes of side 6 cm each joined end to end. Find the surface area of the resulting cuboid.

CH 12-STATISTICS

LESSON PLAN

Class & Section –IX
No of Periods allotted per week: 1
Duration: 40 minutes

Subject: Mathematics
MARCH & APRIL

Unit	Geometry (Chapter 14: Statistics)
Class Transaction	Total: 12 periods (approx. 40 min each)
Pre-requisite for the course	This lesson requires 1. Basic knowledge of The measures of Central Tendency 2. Frequency Distribution and Frequency Tables. 3. Basic Formulae to calculate the given measures.
Assessment of qualifying knowledge	1. Written test 2. Lab activity 3. HW notebook
Objective	• Students would be able to identify the various types of Measures of Central Tendency, their application and calculation. • Students would be also be able to make cumulative frequency curves (ogive), thereby inculcating competencies like **collaboration, critical thinking**
Learning Outcomes	KNOWLEDGE- *Students will know and understand* 1. Various types of Measures of Central Tendency. 2. Different methods to calculate them. 3. Identification and application of suitable method for easy calculations. 4. Identification of the given type of frequency distribution and its corresponding curve. SKILLS- *Students would be able to* 1. Classify data as per the requirement of the situation . 2. Use the given data to interpret the required statistical concept, **creatively** and **critically.** 3. Solve complex questions based on the topics 4. Use **analytical skills** to visualize the given scenario and use the concepts learnt in everyday problems. 5. Use **critical thinking** to solve problems.
Transaction Methodology (The teacher can use the mentioned techniques, wherever applicable, and	Transaction would proceed in the following manner- **Strategy Used: Inductive and deductive reasoning, Brainstorming, graphic organizer, think pair and share** **Think Pair and Share:** In this activity, students think about a particular question individually, then they form pairs to discuss their answers. Following this, the results are shared in a large classroom discussion. Alternatively, two pairs of students can pair up to compare and discuss answers. This process forces students to think individually, and then allows them to analyze and clarify their response collaboratively. **Brain Storming:** In this activity, students are asked to generate ideas on a certain

Sign of Coordinator:

Sign of the Principal:

CHAPTER SUMMARY

What is Statistics?

Statistics is the study of the collection, analysis, interpretation, presentation, and organization of data. In other words, it is a mathematical discipline to collect, summarize data. Also, we can say that statistics is a branch of applied mathematics. However, there are two important and basic ideas involved in statistics; they are uncertainty and variation. The uncertainty and variation in different fields can be determined only through statistical analysis.

Statistics Examples

Some of the real-life examples of statistics are:

- To find the mean of the marks obtained by each student in the class whose strength is 50. The average value here is the statistics of the marks obtained.
- Suppose you need to find how many members are employed in a city. Since the city is populated with 15 lakh people, hence we will take a survey here for 1000 people (sample). Based on that, we will create the data, which is the statistic.

Basics of Statistics

The basics of statistics include the measure of central tendency and the measure of dispersion. The central tendencies are mean, median and mode and dispersions comprise variance and standard deviation.

Mean is the average of the observations. Median is the central value when observations are arranged in order. The mode determines the most frequent observations in a data set.

Variation is the measure of spread out of the collection of data. Standard deviation is the measure of the dispersion of data from the mean. The square of standard deviation is equal to the variance.

Statistics Formulas

The formulas that are commonly used in statistical analysis are given in the table below.

Sample Mean, $\bar{x}$	$\frac{\sum x}{n}$
Population Mean, μ	$\frac{\sum x}{N}$
Sample Standard Deviation, (s)	$\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$
Population Standard Deviation, σ	$\sigma = \sqrt{\frac{(x-\mu)^2}{N}}$
Sample Variance, s^2	$s^2 = \frac{\sum(x_i-\bar{x})^2}{n-1}$
Population Variance, σ^2	$\sigma^2 = \frac{\sum(x_i-\mu)^2}{N}$
Range, (R)	Largest data value – smallest data value

Summary Statistics

In Statistics, summary statistics are a part of descriptive statistics (Which is one of the types of statistics), which gives the list of information about sample data. We know that statistics deals with the presentation of data visually and quantitatively. Thus, summary statistics deals with summarizing the statistical information. Summary statistics generally deal with condensing the data in a simpler form, so that the observer can understand the information at a glance. Generally, statisticians try to describe the observations by finding:

- The measure of central tendency or mean of the locations, such as arithmetic mean.
- The measure of distribution shapes like skewness or kurtosis.
- The measure of dispersion such as the standard mean absolute deviation.
- The measure of statistical dependence such as correlation coefficient.

What is Data in Statistics?

Data is a collection of facts, such as numbers, words, measurements, observations etc.

Types of Data

1. Qualitative data- it is descriptive data.
    - Example- She can run fast, He is thin.
2. Quantitative data- it is numerical information.
    - Example- An Octopus is an Eight legged creature.

Representation of Data

There are different ways to represent data such as through graphs, charts or tables. The general representation of statistical data are:

- Bar Graph
- Pie Chart
- Line Graph
- Pictograph
- Histogram
- Frequency Distribution

Pictorial Representation Using Bar Graph

A bar graph also known as a bar chart is a chart that presents data that is grouped into rectangular bars. Here the length of the bar is directly proportional to the values they represent. The bar graph can be drawn vertically or horizontally. A vertical bar graph is known as a Column Bar Graph. Since one bar graph can be used to display multiple groups of data on the same graph, bar graphs can also be used as comparative tools where the length of the rectangular bar represents the value of each category. Since the rectangular bars are proportional, their differences can be spotted much more easily, visually than through words. Let's take a closer look at bar graphs.

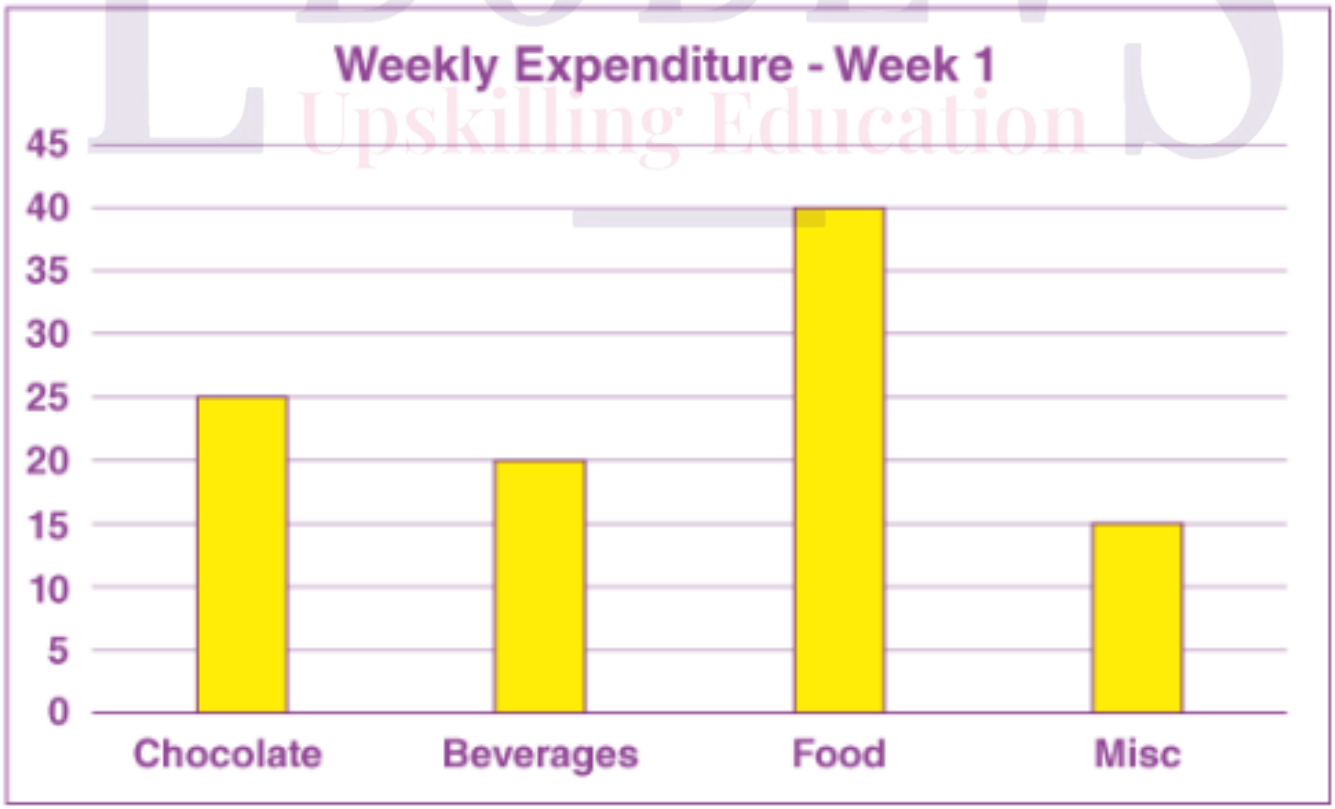
Say you have pocket money of 100 rupees every week. You are allowed to spend this amount any which way you want to. You use this money to buy chocolate, beverages, food and other miscellaneous toys and stuff. What you notice is that every week, the money just seems to disappear. You ask your father for a little more money but he instead suggests that you see where the money is going so that you can learn the value of money. To this end, you grudgingly make a bar chart. But to create a bar chart, you need to have data. You need to note down the things you are spending money on and how much. After a week you have the details of this week's expenditure and they look something like this

Items	Week 1
Chocolate	25
Beverages	20
Food	40
Misc	15

The first thing to observe is how the data is grouped. It is the first step to creating a bar graph. Similar expenses such as chocolates, candies, chewing gums are all grouped together. The same applies to the variety of soft drinks you consume. While discussing a bar graph, it was mentioned that the values are represented as a rectangular bar where the length of the rectangular bar is proportional to the value of the data. Here is where another characteristic of a bar graph comes into play.

Example

A Bar Graph needs to have a uniform scale. The scale dictates the conversion of the data in number into the rectangular format. A bar graph is the representation of numbers using bars of uniform width and length dependent on the number.

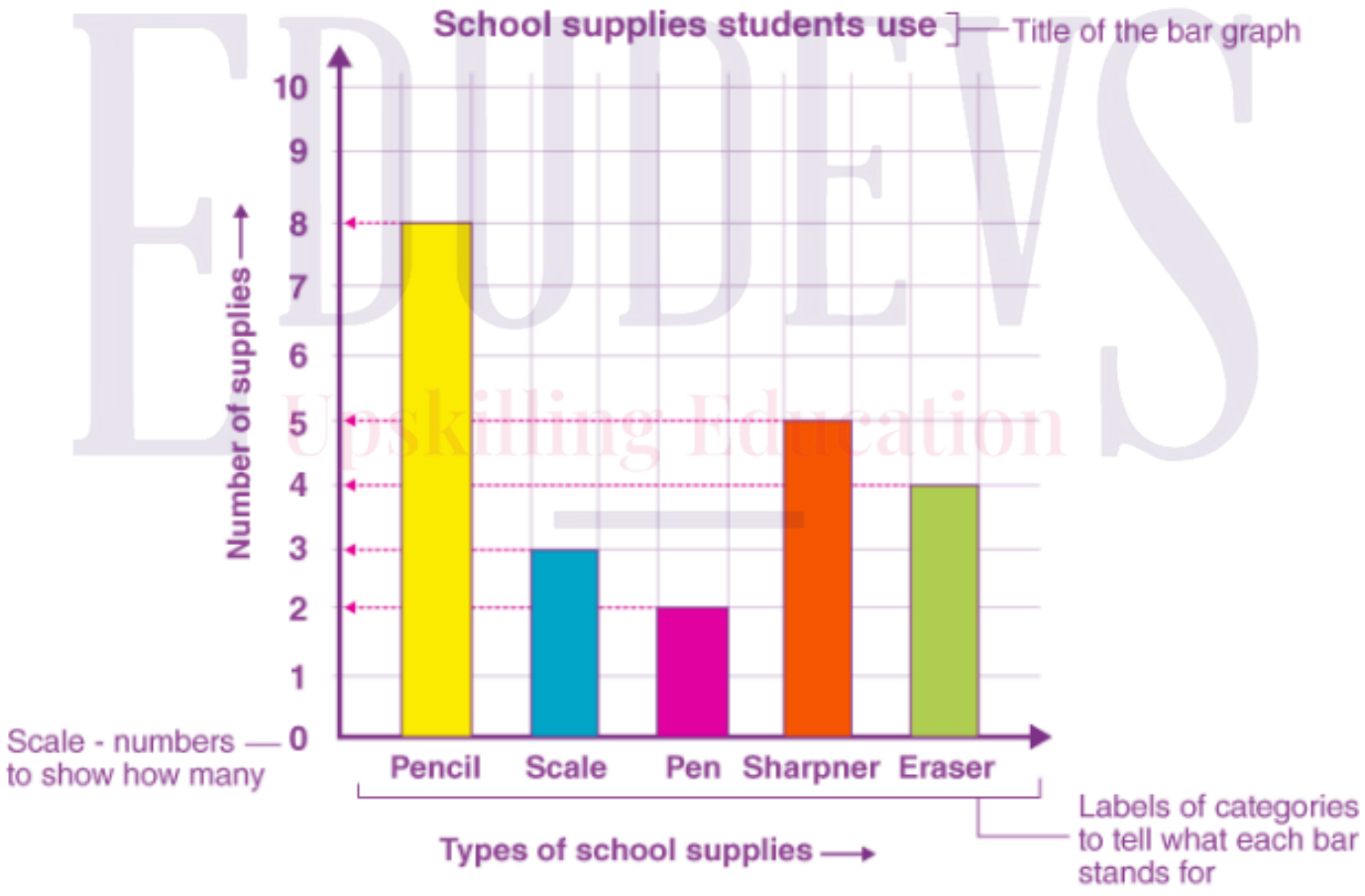


For example, if you represent the money you spent on chocolate using a 25 cm long rectangular bar then the scale is 1 rupee is equal to one unit on the graph i.e. one rupee is represented by one centimeter. But you can clearly see that for this type of representation you will need a massive graph. The thing about scale is that it is completely under our control. So instead of 25 cm, you can represent the same quantity with a rectangular bar of length 25 millimeters and here the scale is 10 rupees is equal to the same one unit i.e. 10 rupees is represented by the same one centimeter.

The interpretation of data is heavily reliant on accurate information about the scale, therefore, it is extremely important to mention the scale of your graph along both the x-axis and y-axis. Using the latter scale i.e. 10 rupees is equal to one centimeter. It is important to remember that the same scale is applied to all the groups of data in the bar graph.

Pictorial Representation of Data using Vertical Bar Graph

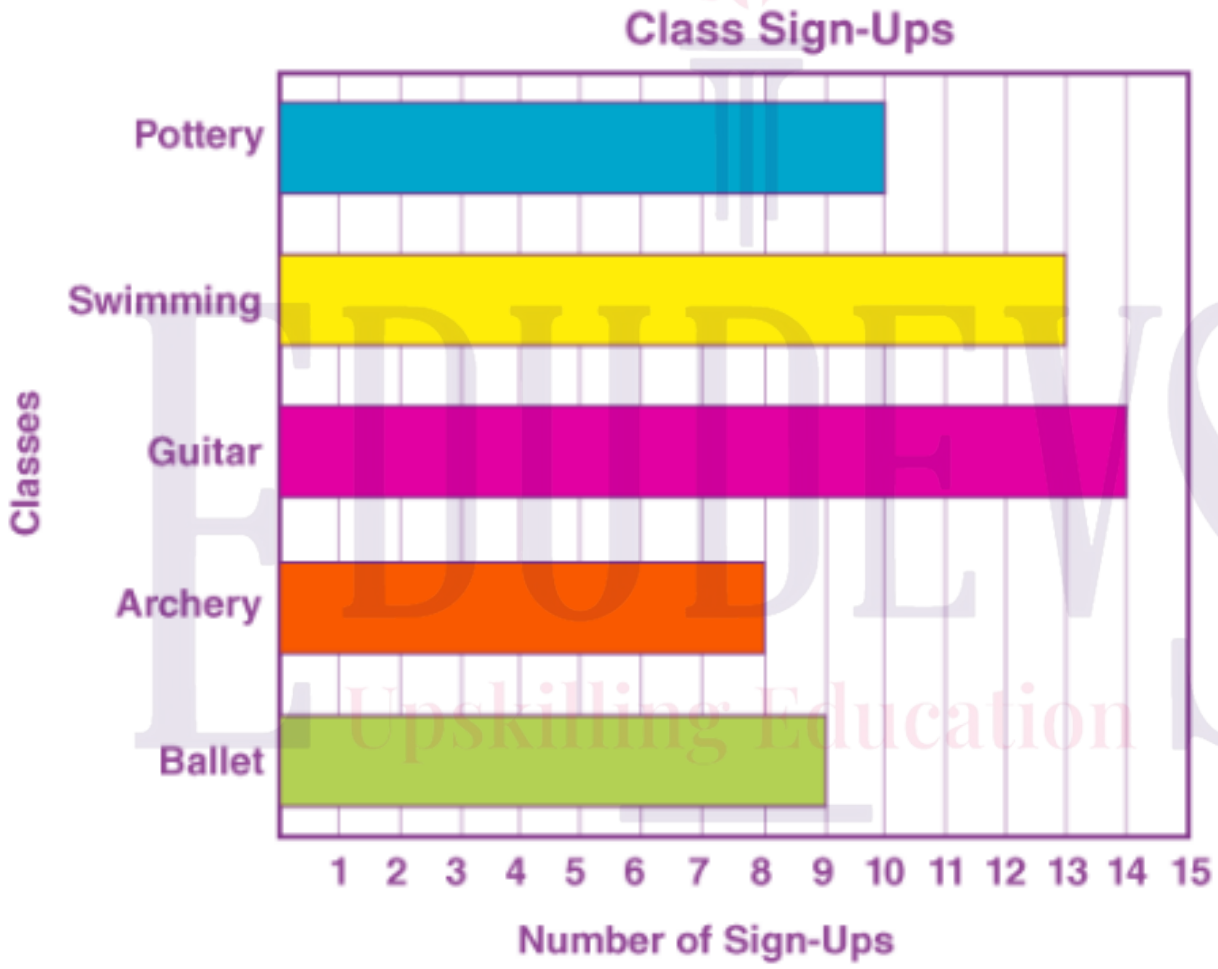
Vertical bar graphs are commonly used pictorial representations to express the given data using vertical bars. Here, the horizontal axis represents the categories and the vertical bars represent the corresponding data for each category. The horizontal axis is the x-axis and the vertical axis is the y-axis.



The vertical bar graphs are also used to represent the series of data and its variation over a period of time. All the vertical bars goes from the bottom of the x-axis to the top.

Pictorial Representation of Data Using Horizontal Bar graph

The horizontal bar graph represents the data using the bars that are parallel to the x-axis. The categories are defined along the y-axis and the respective data are represented using horizontal bars. The bars in the horizontal graph go from left to side along the x-axis.

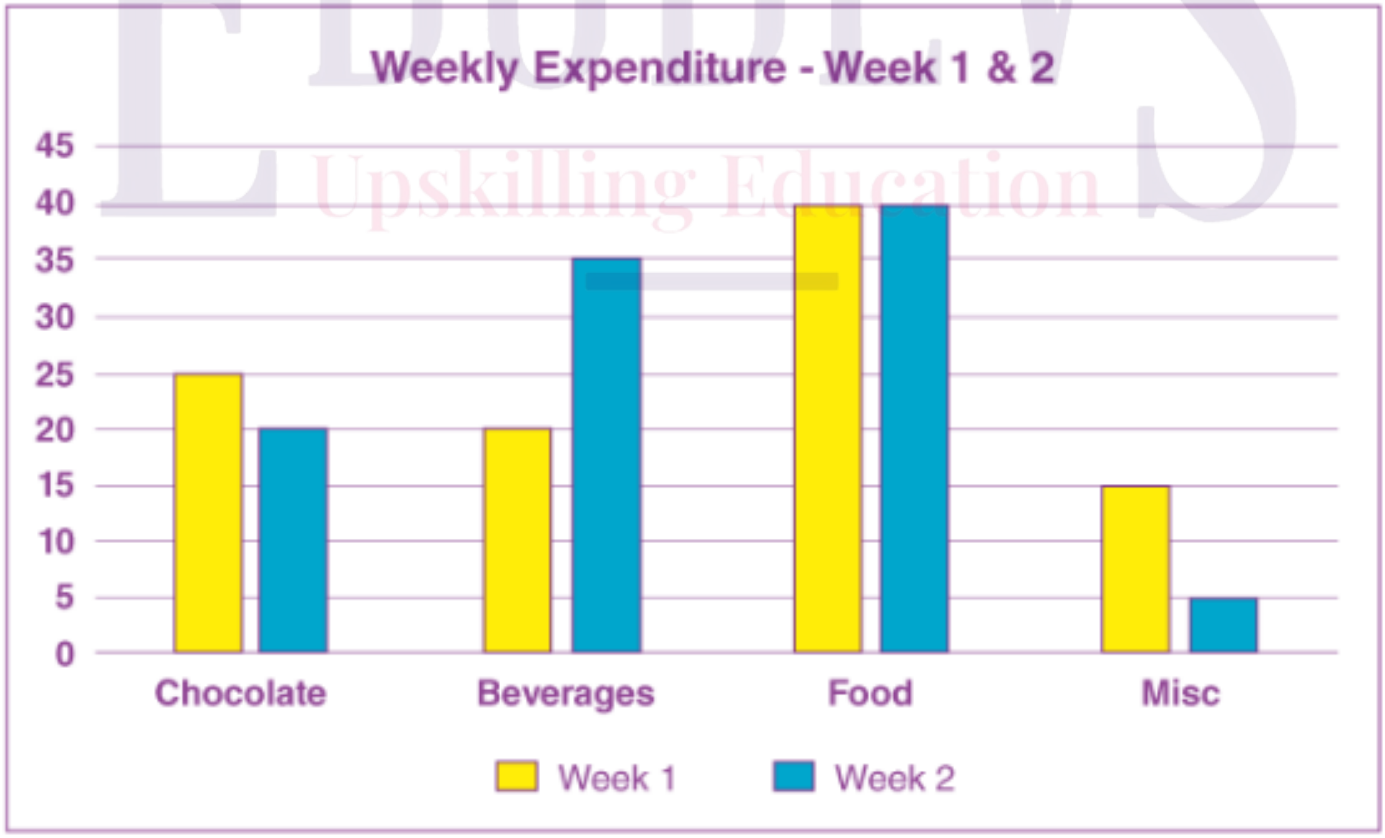


Pictorial Representation of Data Using Double Bar Graph

You have seen what the bar graph of your expenses for the first week looks like. Say you continue this habit of tracking where you spent the money for some more time. You find that your week two expenses are slightly different from your first week. The details of the second-week expenses are;

Items	Week 1	Week 2
Chocolate	25	20
Beverages	20	35
Food	40	40
Misc	15	5

Since they all belong to the same group, they can be charted on the same bar graph. Remember that the same scale applies to all the data in the graph. Instead of charting them individually, you can place similar groups next to each other to compare the changes over a larger time span or over a larger range of data. Now plotting this together, we are better poised to see the differences between the weekly expenditure.



Measures of Central Tendency

The central tendency of the dataset can be found out using the three important measures namely mean,median, &mode

Mean

The mean represents the average value of the dataset. It can be calculated as the sum of all the values in the dataset divided by the number of values. In general, it is considered as the arithmetic mean. Some other measures of mean used to find the central tendency are as follows:

- Geometric Mean
- Harmonic Mean
- Weighted Mean

It is observed that if all the values in the dataset are the same, then all geometric, arithmetic and harmonic mean values are the same. If there is variability in the data, then the mean value differs. Calculating the mean value is completely easy. The formula to calculate the mean value is given by

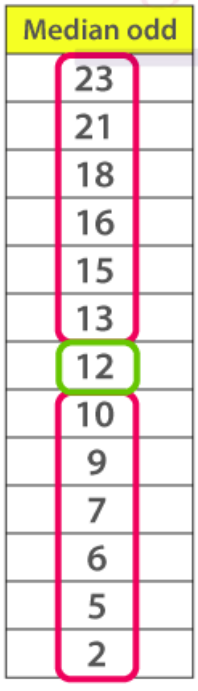
$$Mean = \frac{x_1+x_2+..+x_n}{n}$$

The histogram given below shows that the mean value of symmetric continuous data and the skewed continuous data.

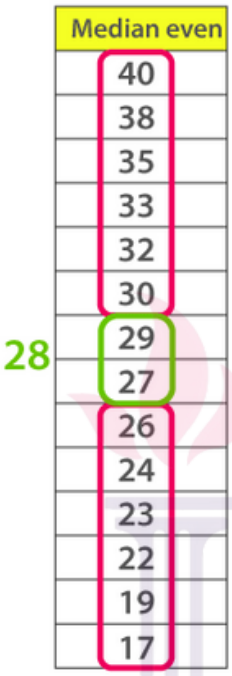
Median

Median is the middle value of the dataset in which the dataset is arranged in the ascending order or in descending order. When the dataset contains an even number of values, then the median value of the dataset can be found by taking the mean of the middle two values.

Consider the given dataset with the odd number of observations arranged in descending order – 23, 21, 18, 16, 15, 13, 12, 10, 9, 7, 6, 5, and 2



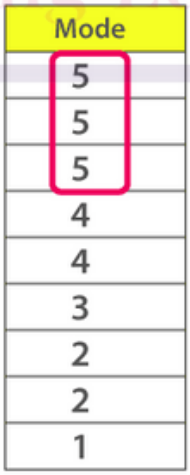
Here 12 is the middle or median number that has 6 values above it and 6 values below it.
Now, consider another example with an even number of observations that are arranged in descending order – 40, 38, 35, 33, 32, 30, 29, 27, 26, 24, 23, 22, 19, and 17



When you look at the given dataset, the two middle values obtained are 27 and 29.
Now, find out the mean value for these two numbers.
i.e.,(27+29)/2 =28
Therefore, the median for the given data distribution is 28.

Mode

The mode represents the frequently occurring value in the dataset. Sometimes the dataset may contain multiple modes and in some cases, it does not contain any mode at all.
Consider the given dataset 5, 4, 2, 3, 2, 1, 5, 4, 5



Since the mode represents the most common value. Hence, the most frequently repeated value in the given dataset is 5.

Solved Examples

1. Give five examples of data that you can collect from your day-to-day life.

Five examples from day-to-day life are
1. The number of students in our class
2. The number of fans in our school
3. Electricity bills of our house for the last two years
4. Election results obtained from television or newspapers
5. Literacy rate figures obtained from Educational Survey

2. The blood groups of 30 students of Class VIII are recorded as follows.
A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.
Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

Solution:

Frequency is the number of students having the same blood group. The frequency is represented in the table or the frequency distribution table.

Blood Group	Number of Students (Frequency)
A	9
B	6
O	12
AB	3
Total	30

The most common Blood Group is the blood group with the highest frequency: O
The rarest Blood Group is the blood group with the lowest frequency: AB

3. The distance (in km) of 40 engineers from their residence to their place of work was found as follows:
5 3 10 20 25 11 13 7 12 31
19 10 12 17 18 11 32 17 16 2
7 9 7 8 3 5 12 15 18 3
12 14 2 9 6 15 15 7 6 12
Construct a grouped frequency distribution table with class size 5 for the data given above, taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

Solution:
Since the given data is very large, we construct a grouped frequency distribution table of class size 5. ∴, class intervals will be 0-5, 5-10, 10-15, 15-20 and so on. The data is represented in the grouped frequency distribution table as

Distances (in km)	Tally marks	Frequency
0 - 5	𝍸	5
5 - 10	𝍸 𝍸 |	11
10 - 15	𝍸 𝍸 |	11
15 - 20	𝍸 ||||	9
20 - 25	|	1
25 - 30	|	1
30 - 35	||	2
Total		40

In the given table, the classes do not overlap. Also, we find that the houses of 36 out of 40 engineers are below 20 km of distance.

4.Find the mean salary of 60 workers in a factory from the following table.

Salary in ₹	Number of workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

Salary (x_i)	Number of workers (f_i)	f_ix_i
3000	16	48000
4000	12	48000
5000	10	50000
6000	8	48000
7000	6	42000
8000	4	32000
9000	3	27000
10000	1	10000
Total	$\Sigma f_i = 60$	$\Sigma f_ix_i = 305000$

$$\bar{x}(\text{Mean}) = \frac{\Sigma fixi}{\Sigma fi} = \frac{305000}{60} = ₹\ 5083.33$$

The mean salary is ₹5083.33

5. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

Solution:
Mode
To find the mode, we first arrange the given data in ascending order.
14,14,14,14,17,18,18,18,22,23,25,28
Here,
We find that 14 occurs most frequently (4 times).
∴ Mode = 14

6. In a mathematics test given to 15 students, the following marks (out of 100) are recorded.
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data.

Solution:
Mean=Average = Sum of all the observations/Total number of observations
= (41+39+48+52+46+62+54+40+96+52+98+40+42+52+60)/15
= 822/15
= 54.8

Median
To find the median, we first arrange the data in ascending order.
39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98
Here,
Number of observations (n) = 15

Since the number of observations is odd, the median can be calculated as
Median = [(n+1)/2]th observation
= [(15+1)/2]th observation
= (16/2)th observation
= 8th observation
= 52

Mode
To find the mode, we first arrange the data in ascending order.
39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98
Here,
We find that 52 occurs most frequently (3 times).
∴ Mode = 52

7. The following table gives the distribution of students in two sections according to the marks obtained by them.

Section A		Section B	
Marks	**Frequency**	**Marks**	**Frequency**
0 - 10	3	0 - 10	5
10 - 20	9	10 - 20	19
20 - 30	17	20 - 30	15
30 - 40	12	30 - 40	10
40 - 50	9	40 - 50	1

Represent the marks of the students of both sections on the same graph by two frequency polygons. From the two polygons, compare the performance of the two sections.

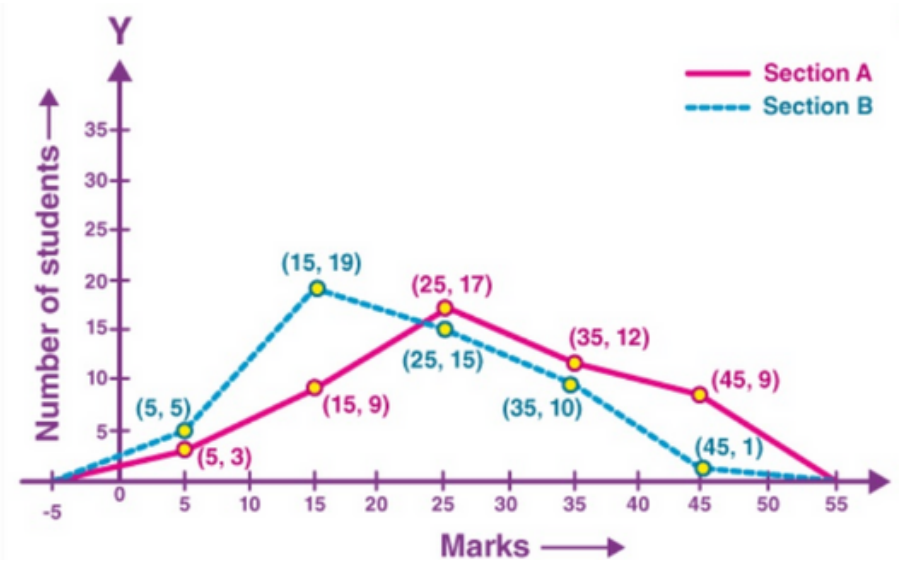
Solution:
The class-marks = (lower limit + upper limit)/2
For section A,

Marks	Class-marks	Frequency
0-10	5	3
10-20	15	9
20-30	25	17
30-40	35	12
40-50	45	9

For section B,

Marks	Class-marks	Frequency
0-10	5	5
10-20	15	19
20-30	25	15
30-40	35	10
40-50	45	1

Representing these data on a graph using two frequency polygon, we get



From the graph, we can conclude that the students of Section A performed better than Section B.

8. A random survey of the number of children of various age groups playing in a park was found as follows:

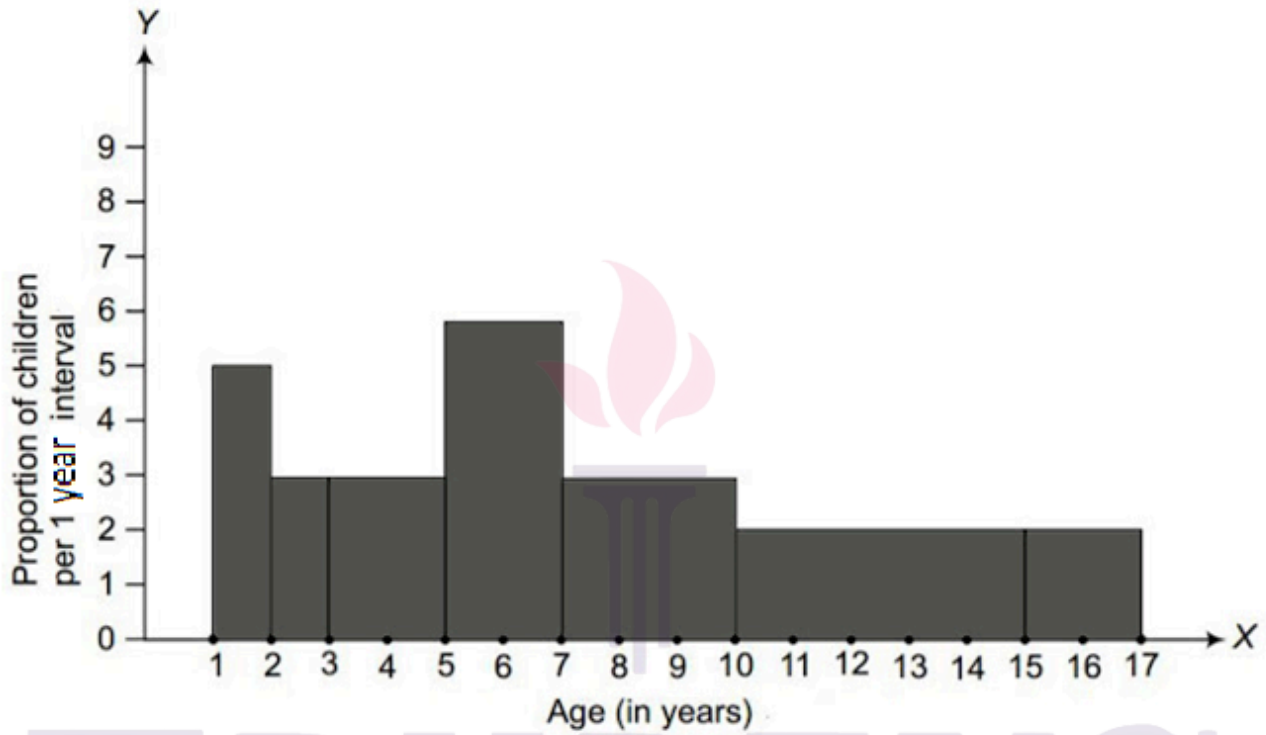
Age (in years)	Number of children
1 - 2	5
2 - 3	3
3 - 5	6
5 - 7	12
7 - 10	9
10 - 15	10
15 - 17	4

Draw a histogram to represent the data above.

Solution:
The width of the class intervals in the given data varies.
We know that,
The area of the rectangle is proportional to the frequencies in the histogram.
Thus, the proportion of children per year can be calculated as given in the table below.

Age (in years)	Number of children (frequency)	Width of class	Length of rectangle
1-2	5	1	(5/1)×1 = 5
2-3	3	1	(3/1)×1 = 3
3-5	6	2	(6/2)×1 = 3
5-7	12	2	(12/2)×1 = 6
7-10	9	3	(9/3)×1 = 3
10-15	10	5	(10/5)×1 = 2
15-17	4	2	(4/2)×1 = 2

Let x-axis = the age of children
y-axis = proportion of children per 1-year interval



ASSIGNMENT FOR CLASS WORK

1. Give one example of a situation in which
(i) the mean is an appropriate measure of central tendency.
(ii) the mean is not an appropriate measure of central tendency, but the median is an appropriate measure of central tendency.

2. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.
29, 32, 48, 50, x, x+2, 72, 78, 84, 95

3. The following number of goals were scored by a team in a series of 10 matches:
2, 3, 4, 5, 0, 1, 3, 3, 4, 3
Find the mean, median and mode of these scores.

4. 100 surnames were randomly picked up from a local telephone directory, and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1 - 4	6
4 - 6	30
6 - 8	44
8 - 12	16
12 - 20	4

(i) Draw a histogram to depict the given information.
(ii) Write the class interval in which the maximum number of surnames lie.

5. Given below are the seats won by different political parties in the polling outcome of state assembly elections:

Political party	A	B	C	D	E	F
Seats won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.
(ii) Which political party won the maximum number of seats?

6. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 – 44 (in years) worldwide found the following figures (in %):

S.No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

7.A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:
2.6 3.0 3.7 3.2 2.2 4.1 3.5 4.5
3.5 2.3 3.2 3.4 3.8 3.2 4.6 3.7
2.5 4.4 3.4 3.3 2.9 3.0 4.3 2.8
3.5 3.2 3.9 3.2 3.2 3.1 3.7 3.4
4.6 3.8 3.2 2.6 3.5 4.2 2.9 3.6
Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from interval 2 – 2.5.

8. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:
1 6 2 3 5 12 5 8 4 8
10 3 4 12 2 8 15 1 17 6
3 2 8 5 9 6 8 7 14 12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals 5-10.
(ii) How many children have watched television for 15 or more hours a week?

9. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:
0 1 2 2 1 2 3 1 3 0
1 3 1 1 2 2 0 1 2 1
3 0 0 1 1 2 3 2 2 0
Prepare a frequency distribution table for the data given above.

10. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

(i) Construct a grouped frequency distribution table with classes 84 – 86, 86 – 88, etc.
(ii) Which month or season do you think this data is about?
(iii) What is the range of this data?

WORKSHEET FOR ASSESSMENT

1. **The following number of goals were scored by a team in a series of 10 matches:**
2, 3, 4, 5, 0, 1, 3, 3, 4, 3
Find the mean, median and mode of these scores.

2. In a Mathematics test given to 15 students, the following marks (out of 100) ar recorded.
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data.

3. Following data gives the marks (out of 50), obtained by 30 students of a class in test:

21	13	48	1	10	8	12	17	19	17
40	12	46	37	17	27	30	6	12	23
19	39	25	5	33	19	21	12	41	9

4.In a small unit of a factory 5 employees (a supervisor and four labourers) ar working. The laboures draw a salary of Rs. 5000 per month each while the supervis gets Rs. 15,000 per month. Calculate the mean, median and mode of the salaries of th unit of the factory.

5. Find the value of p when the mean of the following data is 21.6.

x_i	6	12	18	24	30	36
f_i	5	4	p	6	4	6

6. Following table gives the distribution of two sections of a class according to the marks obtained by them:

Section A		Section B	
Marks	Frequency	Marks	Frequency
0–10	9	0–10	5
10–20	3	10–20	15
20–30	17	20–30	19
30–40	9	30–40	10
40–50	12	40–50	4

7. A study was conducted to find out the concentration of sulphur dioxide in the air i parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:
0.03 0.08 0.08 0.09 0.04 0.17
0.16 0.05 0.02 0.06 0.18 0.20
0.11 0.08 0.12 0.13 0.22 0.07
0.08 0.01 0.10 0.06 0.09 0.18
0.11 0.07 0.05 0.07 0.01 0.04
(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 – 0.04, 0.04 – 0.08, and so on.
(ii) For how many days was the concentration of Sulphur dioxide more than 0.11 parts per million?

8. A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:
0.03 0.08 0.08 0.09 0.04 0.17
0.16 0.05 0.02 0.06 0.18 0.20
0.11 0.08 0.12 0.13 0.22 0.07
0.08 0.01 0.10 0.06 0.09 0.18
0.11 0.07 0.05 0.07 0.01 0.04
(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 – 0.04, 0.04 – 0.08, and so on.
(ii) For how many days was the concentration of Sulphur dioxide more than 0.11 parts per million?

9.The value of π up to 50 decimal places is given below:
3.14159265358979323846264338327950288419716939937510
(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.
(ii) What are the most and the least frequently occurring digits?

10.Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

CH 15-PROBABILITY

LESSON PLAN

Class & Section –IX
No of Periods allotted per week: 1
Duration: 40 minutes

Subject: Mathematics
MARCH & APRIL

Class	Subject	Time Slot	Lesson	Topic P1
IX	MATHS	35 Mins 1 pd	Probability	Introduction

Expected Learning Outcome :The learners will be able to :

- Define and give meaning of probability
- Correlated probability with real life situation.

Expected skill development:

- Developing connections among different terms like chances, events, trails, sample space etc.
- Different thinking skills while listing total outcomes and identifying favourable outcomes
- Exhibiting calculation skills and procedural thinking and finding probability
- Analytical thinking
- Problem solving.

FLOW OF LESSON WITH RESPECT TO TIME (Introduction/warm-up, Content, Think Line, Closure)

Pedagogy	Content
(10 +5 Min) Warm Up **Demonstration by Fecililator** In the very first class Facilitator will discuss about the statements that come across everyday life.	1)It will probably rain today 2)I doubt that he will pass the test 3)Chances are high that the price of petrol will go up Facilitator will ask students to give examples The words most probably, chances, doubt etc. show uncertainty or probability of occurrence of an event. It is used in science, commerce, biological sciences, weather forecasting etc. Many books are there on probability. These days probability theory is extensively used in various fields.

Sign of Coordinator:

Sign of the Principal:

CHAPTER SUMMARY

Introduction to Probability

Probability

- Probability is the measure of the likelihood of an event to occur. Events can't be predicted with certainty but can be expressed as to how likely it can occur using the idea of probability.
- Probability can range between 0 and 1, where 0 probability means the event is an impossible one and probability of 1 indicates a certain event.

For example, when we toss a coin, either we get Head OR Tail, only two possible outcomes are possible (H, T). But when two coins are tossed then there will be four possible outcomes, i.e {(H, H), (H, T), (T, H), (T, T)}.

Formula for Probability

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favourable outcomes and the total number of outcomes.

Probability of event to happen P(E) = Number of favourable outcomes/Total Number of outcomes

Solved Examples

1) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

Sol:
The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

2There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

Sol:

- No. of blue bottles picked out: 300
- No. of red bottles: 200
- No. of green bottles: 450
- No. of orange bottles: 50

a) What is the probability that Sumit will pick a green bottle?

For every 1000 bottles picked out, 450 are green.
Therefore, P(green) = 450/1000 = 0.45

b) If there are 100 bottles in the container, how many of them are likely to be green?

The experiment implies that 450 out of 1000 bottles are green.
Therefore, out of 100 bottles, 45 are green.

Probability of an Event

Assume an event E can occur in r ways out of a sum of n probable or possible equally likely ways. Then the probability of happening of the event or its success is expressed as;

$P(E) = r/n$

The probability that the event will not occur or known as its failure is expressed as:

$P(E') = (n-r)/n = 1-(r/n)$

E' represents that the event will not occur.
Therefore, now we can say;

$P(E) + P(E') = 1$

This means that the total of all the probabilities in any random test or experiment is equal to 1.

What are Equally Likely Events?

When the events have the same theoretical probability of happening, then they are called equally likely events. The results of a sample space are called equally likely if all of them have the same probability of occurring. For example, if you throw a die, then the probability of getting 1 is 1/6. Similarly, the probability of getting all the numbers from 2,3,4,5 and 6, one at a time is 1/6.

Hence, the following are some examples of equally likely events when throwing a die:
Getting 3 and 5 on throwing a die
Getting an even number and an odd number on a die
Getting 1, 2 or 3 on rolling a die
are equally likely events, since the probabilities of each event are equal.

Complementary Events

The possibility that there will be only two outcomes which states that an event will occur or not. Like a person will come or not come to your house, getting a job or not getting a job, etc. are examples of complementary events. Basically, the complement of an event occurring in the exact opposite that the probability of it is not occurring. Some more examples are:

- It will rain or not rain today
- The student will pass the exam or not pass.
- You win the lottery or you don't.

3.Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Solution:
Number of times 2 heads come up = 72
Total number of times the coins were tossed = 200
∴, the probability of 2 heads coming up = 72/200 = 9/25

4.Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):
4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Solution:
Total number of bags present = 11
Number of bags containing more than 5 kg of flour = 7
∴, the probability that any of the bags chosen at random contains more than 5 kg of flour = 7/11

Examples

1.Find the probability of 'getting 3 on rolling a die'.

Solution:

Sample Space = S = {1, 2, 3, 4, 5, 6}
Total number of outcomes = n(S) = 6
Let A be the event of getting 3.
Number of favourable outcomes = n(A) = 1
i.e. A = {3}
Probability, P(A) = n(A)/n(S) = 1/6
Hence, P(getting 3 on rolling a die) = 1/6

2: Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

Solution:
A standard deck has 52 cards.
Total number of outcomes = n(S) = 52

Let E be the event of drawing a face card.

Number of favourable events = n(E) = 4 x 3 = 12 (considered Jack, Queen and King only)

Probability, P = Number of Favourable Outcomes/Total Number of Outcomes
P(E) = n(E)/n(S)
= 12/52
= 3/13
P(the card drawn is a face card) = 3/13

3. A vessel contains 4 blue balls, 5 red balls and 11 white balls. If three balls are drawn from the vessel at random, what is the probability that the first ball is red, the second ball is blue, and the third ball is white?

Solution:
Given,
The probability to get the first ball is red or the first event is 5/20.
Since we have drawn a ball for the first event to occur, then the number of possibilities left for the second event to occur is 20 – 1 = 19.
Hence, the probability of getting the second ball as blue or the second event is 4/19.
Again with the first and second event occurring, the number of possibilities left for the third event to occur is 19 – 1 = 18.
And the probability of the third ball is white or the third event is 11/18.
Therefore, the probability is 5/20 x 4/19 x 11/18 = 44/1368 = 0.032.
Or we can express it as: P = 3.2%.

4. Two dice are rolled, find the probability that the sum is:

1. **equal to 1**
2. **equal to 4**
3. **less than 13**

Solution:
To find the probability that the sum is equal to 1 we have to first determine the sample space S of two dice as shown below.
S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }
So, n(S) = 36
1) Let E be the event “sum equal to 1”. Since, there are no outcomes which where a sum is equal to 1, hence,
P(E) = n(E) / n(S) = 0 / 36 = 0

2) Let A be the event of getting the sum of numbers on dice equal to 4.
Three possible outcomes give a sum equal to 4 they are:
A = {(1,3),(2,2),(3,1)}
n(A) = 3
Hence, P(A) = n(A) / n(S) = 3 / 36 = 1 / 12

3) Let B be the event of getting the sum of numbers on dice is less than 13.
From the sample space, we can see all possible outcomes for the event B, which gives a sum less than B. Like:
(1,1) or (1,6) or (2,6) or (6,6).
So you can see the limit of an event to occur is when both dies have number 6, i.e. (6,6).
Thus, n(B) = 36
Hence,
P(B) = n(B) / n(S) = 36 / 36 = 1

Experimental Probability Example

You asked your 3 friends Shakshi, Shreya and Ravi to toss a fair coin 15 times each in a row and the outcome of this experiment is given as below:

Coin Tossed By:	No. of. Heads	No. of. Tails
Shakshi	6	9
Shreya	7	8
Ravi	8	7

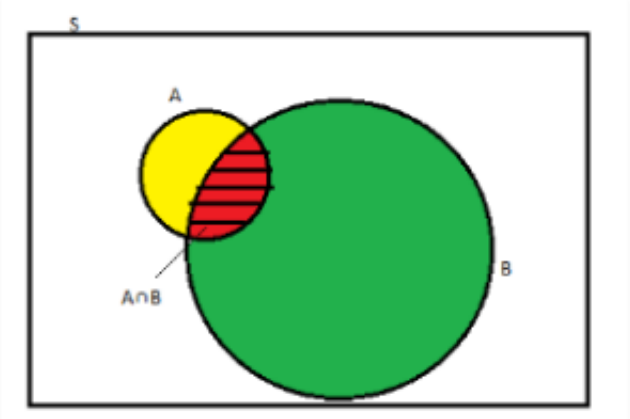
Calculate the probability of occurrence of heads and tails.
Solution: The experimental probability for the occurrence of heads and tails in this experiment can be calculated as:
Experimental Probability of Occurrence of heads = Number of times head occurs/Number of times coin is tossed.
Experimental Probability of Occurrence of tails = Number of times tails occurs/Number of times coin is tossed.

Coin Tossed By:	No. of. Heads	No. of. Tails	Experimental Probability for the occurrence of Head	Experimental Probability for the occurrence of Tail
Shakshi	6	9	6/15 = 0.4	9/15 = 0.6
Shreya	7	8	7/15 = 0.47	8/15 = 0.53
Ravi	8	7	8/15 = 0.53	7/15 = 0.47

We observe that if the number of tosses of the coin increases then the probability of occurrence of heads or tails also approaches to 0.5.

Conditional Probability Definition

The probability of occurrence of any event A when another event B in relation to A has already occurred is known as conditional probability. It is depicted by P(A|B).



Mathematically this can be represented as,
P(A|B) = N(A∩B)/N(B)

Where P(A|B) represents the probability of occurrence of A given B has occurred.
N(A ∩ B) is the number of elements common to both A and B.
N(B) is the number of elements in B and it cannot be equal to zero.
Let N represent the total number of elements in the sample space.

$$\Rightarrow P(A|B) = \frac{\frac{N(A \cap B)}{N}}{\frac{N(B)}{N}}$$

Since N(A ∩ B)/N and N(B)/N denotes the ratio of the number of favourable outcomes to the total number of outcomes, therefore, it indicates the probability.
Therefore, N(A ∩ B)/N can be written as P(A ∩ B) and N(B)/N as P(B).
⇒ P(A|B) = P(A ∩ B)/P(B)
Therefore, P(A ∩ B) = P(B) P(A|B) if P(B) ≠ 0
= P(A) P(B|A) if P(A) ≠ 0
Similarly, the probability of occurrence of B when A has already occurred is given by,
P(B|A) = P(B ∩ A)/P(A)

Conditional Probability Example

Example: Two dies are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once?

Solution:
The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6 i.e. 36 events.
Event A indicates the combination in which 3 has appeared at least once.
Event B indicates the combination of the numbers which sum up to 7.
A = {(3, 1), (3, 2), (3, 3)(3, 4)(3, 5)(3, 6)(1, 3)(2, 3)(4, 3)(5, 3)(6, 3)}
B = {(1, 6)(2, 5)(3, 4)(4, 3)(5, 2)(6, 1)}
P(A) = 11/36
P(B) = 6/36
A ∩ B = 2
P(A ∩ B) = 2/36
Applying the conditional probability formula we get,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

ASSIGNMENT FOR CLASSWORK

1. **In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.**

2.1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

1. **(i) 2 girls (ii) 1 girl (iii) No girl**
2. **Also check whether the sum of these probabilities is 1.**

3.An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

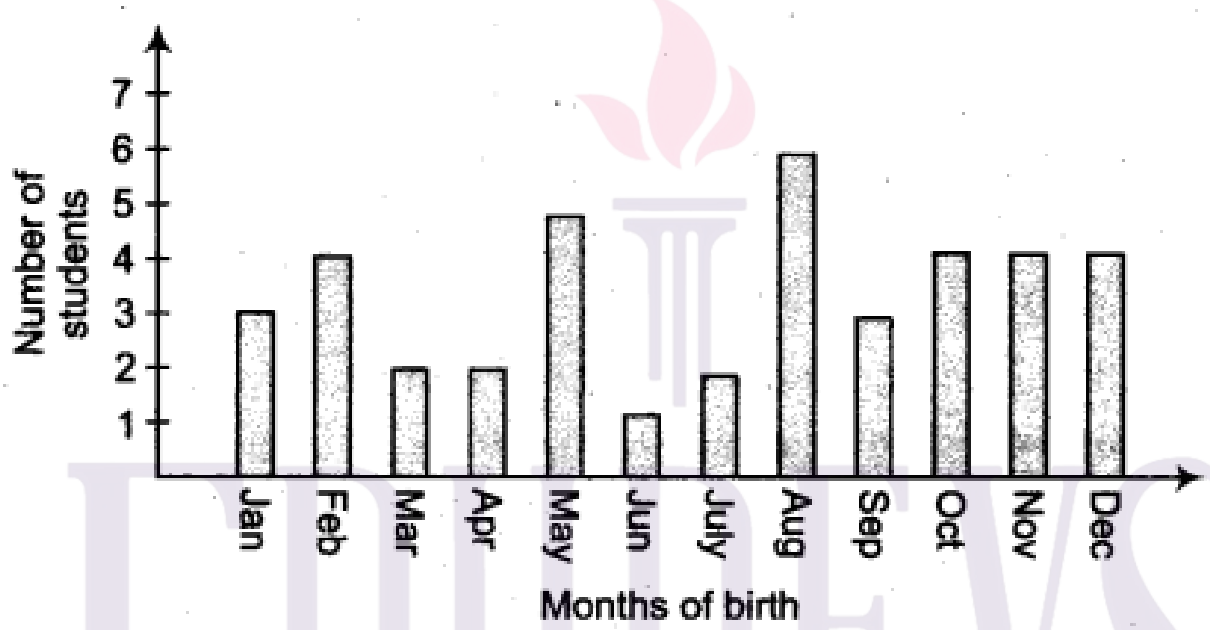
Suppose a family is chosen. Find the probability that the family chosen is
(i) earning ₹10000 – 13000 per month and owning exactly 2 vehicles.
(ii) earning ₹16000 or more per month and owning exactly 1 vehicle.
(iii) earning less than ₹7000 per month and does not own any vehicle.
(iv) earning ₹13000 – 16000 per month and owning more than 2 vehicles.
(v) owning not more than 1 vehicle.

ASSIGNMENT FOR CLASSWORK

4. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays Find the probability that she did not hit a boundary.

5.An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

6.In a particular section of class IX, 40 students were asked about the month of their birth and the following graph was prepared for the data so obtained.



Find the probability that a student of the class was born in August.

7.In a particular section of class IX, 40 students were asked about the month of their birth and the following graph was prepared for the data so obtained.

8.To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random
(i) likes statistics, (ii) does not like it.

WORKSHEET FOR ASSESSMENT

1. **One card is drawn from a well shuffled deck of 52 cards. Find the probability that the number on it is a prime number.**

2. The following table gives distance (in km) that 40 engineers have to travel fro their residences to their work places:-

Distance (in km)	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35
No of engineers	5	11	11	9	1	1	2

Find the probability that an engineer selected at random lives at a distance of:-
(i) 10 – 15 km (event E1) (ii) more than 35 km (event E2) (iii) less than 10 km (ever E3) (iv) upto 35 km (event E4)

3. Blood groups of 30 students in a class are as follows:

Blood group	O^+	O^-	A^+	A^-	B^+	B^-	AB^+	AB^-
Total	8	2	6	1	10	2	1	0

Find the probability that a student selected at random has a blood group: - (i) O (ii) B + (iii) AB

4.Weekly wages of workers in a factory are as follows:-

Weekly wages (in Rs)	325 - 350	350 - 375	375 - 400	400 - 425	425 – 450
No of workers	0	45	75	60	40

Find the probability that a worker selected at random earns: - (i) Rs 400 or more
(ii) Rs 375 or more but less than Rs 425
(iii) Upto Rs 400 (iv) at least Rs 375

5. Given below are the seats won by different political parties in the polling outcome a state assembly elections:-

Political party	A	B	C	D	E	F
Seats won	75	55	37	29	10	37

Find the probability that any of the political parties chosen has won more than 30 seats.

6. A batsman's score in 80 ODI's is as follows:-

Scores	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89	90 - 99
No of matches	1	1	8	13	20	22	12	3

What is the probability that the batsman will score (in the next match)
(i) atleast 70 runs (ii) less than 50 runs (iii) 40 to 69 runs (iv) at most 59 runs

7. Three coins are tossed simultaneously 200 times with the following frequencies o different outcomes:

Outcomes	Frequency
3 heads	23
2 heads	72
1 head	77
No head	28

Find the probability of getting (i) 3 heads (ii) 2 heads & 1 tail (iii) at least 2 heads (iv) getting more tails than heads

8. The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	% of marks obtained
I	58
II	64
III	76
IV	62
V	85

Find the probability that the student gets
(i) a first class i.e. at least 60% marks (iii) a distinction i.e. 75% or above
(ii) marks between 70% and 80% (iv) less than 65% marks

9. Following frequency distribution gives the weights of 38 students of a class

Weight (in kg)	Number of students
31 – 35	9
36 – 40	5
41 – 45	14
46 – 50	3
51 – 55	1
56 – 60	2
61 – 65	2
66 – 70	1
71 – 75	1

Find the probability that weight of a student in the class is:
(i) At most 60 kg (ii) at least 36 kg (iii) not more than 50 kg

10. Define two events each, one having probability 0 and the other having probability of 1.

11.In n trials of a random experiment if an event A occurs m times, event B occurs times and event C occurs r times, s.t m+p+r = n
then find the value of: (i) P (A) (ii) P (B) (iii) P (C) (iv) P (A) + P (B) + P (C) (v) P (not C)
(vi) P (neither A nor C) (VII) P (both B and C)